Balassa–Samuelson in the Long Run: Qualitative Success, Quantitative Limits*

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Abstract

I qualitatively and quantitatively revisit the Balassa–Samuelson (BS) mechanism in the long run. Traditional panel regression specifications without time fixed effects are fragile, but adding time fixed effects yields a stable, positive BS elasticity across samples and frequencies—evidence that the data support BS qualitatively on average across countries. Quantitatively, however, a standard multi-country trade model fed only by observed sectoral productivity cannot match country paths and delivers too small magnitudes. These failures persist with costly trade, multi-country, multi-sector settings, input–output linkages, and time-varying trade costs.

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1 Introduction

One of the most influential ideas in international economics, developed by Roy Harrod and later formalized by Béla Balassa and Paul Samuelson, is that countries experiencing faster productivity growth in tradables relative to non-tradables should see their price levels appreciate in real terms. Despite its intuitive appeal, the empirical record is mixed.[1]

This paper qualitatively and quantitatively analyzes how the Balassa–Samuelson effect explains the evolution of RERs across countries. Qualitatively, I show that the traditional specification is not robust. I show that the traditional specification requires the number of time periods T to go to ∞, and that letting the number of countries N go to ∞ does not guarantee consistency. This helps rationalize the mixed evidence in the literature across settings, in particular across reference countries. I retain a single-reference setting and add time fixed effects that absorb period-common movements, yielding consistency under standard exogeneity and a $\sqrt{NT}$ (rather than $\sqrt{T}$) rate that stabilizes the estimates. I show that, on average, the Balassa–Samuelson effect is qualitatively present across different countries, sample periods, time frequencies, datasets, and labor-productivity measures.

Quantitatively, I develop a standard trade model and evaluate how much sectoral productivity growth can explain RERs. Feeding observed productivity growth does not replicate RER paths quantitatively, and it fails even qualitatively in some countries. This quantitative failure persists after introducing costly trade, multiple countries, input–output linkages, and time-varying trade costs.

Empirical Analysis In the first half of the paper, I sharpen the standard test of the Balassa–Samuelson (BS) mechanism. I first show that traditional single-reference panels *without* time effects perform poorly, even on average. Estimates are fragile with respect to the chosen numeraire: switching the reference country moves the coefficients substantially—only a few benchmarks yield significant positive estimates, while others are imprecise or even wrong-signed. I show econometrically that consistency of the traditional specification requires the number of periods $T \to \infty$, and that letting the number of countries $N \to \infty$ does not guarantee consistency. In particular, without $T \to \infty$, the estimates are not consistent.

I then retain a single reference but add time fixed effects. I show that time effects

[1] See Tica and Družić (2006) for reviews and Berka and Steenkamp (2018) and Berka et al. (2018) for more recent evidence.

absorb period-common movements and shift identification to within-year cross-sectional differences; under standard exogeneity, the estimator is consistent and converges at the $\sqrt{NT}$ rate rather than $\sqrt{T}$. With time fixed effects, the BS pattern becomes visible: the relative tradable–non-tradable productivity ratio is positively associated with real exchange rates, and the corresponding estimates are tighter. These findings are robust across countries, sample periods, time frequencies, filtering choices, specifications, and labor-productivity measures.

Quantitative Analysis In the second half of the paper, I move from reduced form to a general equilibrium environment. I feed the same sectoral series $\{A_{i,s,t}\}$—and, when relevant, bilateral iceberg costs $\{\tau_{i,j,s,t}\}$—into five settings: (i) a two-country, two-sector free-trade case to isolate the textbook BS channel, (ii) a two-country, two-sector costly-trade case to quantify attenuation from frictions, (iii) a multi-country, two-sector costly-trade case that allows geography and partner reallocation, (iv) a multi-country, two-sector costly-trade case with input-output linkages, and (v) a multi-country, multi-sector costly-trade case.[2] In all experiments, I solve the static equilibrium year by year, compute sectoral price indices and the CPI, and compare model-implied real effective exchange rates (REER, trade-weighted real exchange rates) to the data.

Feeding only sectoral productivity shocks—the classical BS channel—performs poorly. Across all environments, model-implied REER movements are a fraction of those in the data, and for several countries, even the sign is wrong. The quantitative shortfall persists when moving from the simple textbook case to the full multi-country, multi-sector setting.

Allowing time-varying trade costs to move along their inferred low-frequency paths does not change the conclusion. Adding $\{\tau_{i,j,s,t}\}$ on top of $\{A_{i,s,t}\}$ leaves the slope of fitted versus actual REER close to zero in long differences, with only marginal improvements in a few cases. Incorporating input–output linkages yields the same result: the BS mechanism remains quantitatively too weak.

At the sectoral level, service-sector prices co-move with sectoral productivity in the expected direction and align somewhat better than the aggregate CPI, but magnitudes are still far too small. Goods-sector prices fare no better. These patterns remain unchanged when both productivity and trade-cost shocks are combined.

I conclude with suggestive evidence that the failure comes from the inability of the models to explain cross-country wages. I show that the models cannot account for changes

[2]See Davis and Weinstein (2001) for the exercises in the same spirit in the context of the Heckscher-Ohlin-Vanek theory.

in wages and that directly feeding wages to the model can almost perfectly predict changes in REERs.[3]

Related Literature This paper contributes to two strands of the literature. First, I contribute to the empirical literature that investigates whether the Balassa–Samuelson mechanism—that countries with faster productivity growth in tradable industries experience real appreciation—holds in the data. The empirical record so far is mixed across specifications, time periods, and country samples.[4] Some studies find a positive association between relative productivity and real exchange rates (Officer, 1976; Hsieh, 1982; Lee and Tang, 2007; Lothian and Taylor, 2008; Cardi and Restout, 2015; Devereux et al., 2025), while others do not (Canzoneri et al., 1999; Berka et al., 2018; Berka and Steenkamp, 2018). Notably, Berka et al. (2018); Berka and Steenkamp (2018) include unit labor costs as covariates to restore the theory-consistent sign on the relative productivity term. [5] Other studies shift focus from aggregate real exchange rates to sectoral prices and show that relative sectoral productivity can empirically explain differences in service (or non-tradable) prices (De Gregorio et al., 1994; Canzoneri et al., 1999; Devereux et al., 2025).[6]

My contribution is to show that, contrary to the existing literature, the Balassa–Samuelson mechanism is, on average, qualitatively valid without these adjustments and robust across specifications, time periods, countries in the sample, time frequencies, and productivity measures. I demonstrate that previous regressions using a single reference country suffer from an over-weighting of that reference country and are not consistent. As I discuss, without time fixed effects, their econometric designs are fragile and can generate spurious support that depends on the chosen reference country and its error structure. In a within-Europe setting, which typically supports BS, this fragility can be masked, so evidence for BS in panels without time fixed effects may reflect a statistical coincidence rather than robust identification.

Second, I contribute to the quantitative literature that explores structural drivers behind the time paths of real exchange rates. Most quantitative studies focus on short- or medium-run dynamics of the RER (Berka et al., 2018; Chahrour et al., 2024; Gornemann

[3]Nevertheless, wages are endogenous equilibrium objects. Thus, my finding simply shows that the failure comes from the model's inability to explain wages, not from issues in the CES price aggregation systems.

[4]See Froot and Rogoff (1995); Tica and Družić (2006) for reviews.

[5]Their main specifications control unit labor costs, which are the endogenous objects, depending on productivity. Thus, their estimates of the coefficients on the productivity term reflect residual effects of productivity on real exchange rates, keeping wages constant, instead of the Balassa-Samuelson effect itself.

[6]See Engel (1999), who shows that real exchange rates and non-tradable prices are disconnected. See also Ito et al. (1999) and Hassan (2016), focusing on aggregate productivity, which discuss the possibility of higher applicability to middle to high-income countries.

et al., 2025).[7] A few exceptions include Irwin and Obstfeld (2024) and Devereux et al. (2025). Irwin and Obstfeld (2024) decompose the real exchange rate into several price components and show that the relative sectoral price is important in explaining the depreciation of South Korea.[8] A closely related paper is Devereux et al. (2025), which examines the drivers behind stable real exchange rates of Eastern European countries against the average European country between 1999 and 2020.

My quantitative part differs from Devereux et al. (2025) in three dimensions. First, I explicitly focus on long-run *changes* in RERs and sectoral productivity using low-frequency (long-difference) variation, whereas they evaluate the model year by year.[9] Their approach mixes trend and cyclical movements; ours is designed to speak to low-frequency change. Second, I cover a broader set of countries; by contrast, they focus on Eastern Europe relative to Western Europe as a bloc. While they report country-level results and track some cases (e.g., Slovakia) reasonably well, performance is weak for many other countries. Third, I compare multiple models, including N-country environments with IO linkages, whereas they use a two-country tradable–non-tradable setup with cross-sector intermediate linkages. I show that such 2×2 environments can overstate the dispersion of counterfactual outcomes relative to N-country versions, though both models fail to explain the evolution of RERs at the country-level.

Roadmap. Section 2 presents the theoretical motivation, data, and empirical specifications. I first replicate traditional benchmark-based regressions and econometrically show why results hinge on the chosen reference. I then estimate the model with time fixed effects across various settings. Section 3 develops the quantitative model. Section 4 feeds observed productivity and trade costs into multiple environments and compares model-implied REER to the data. Section 5 concludes.

2 Empirical Analysis

2.1 Basic Specification for Testing Balassa–Samuelson Effects

Theoretical Motivation Consider a two-country (i and j) and two-sector ($s = T, NT$) setup following the classical Balassa–Samuelson framework. Each country produces a

[7] See Itskhoki (2021) for more comprehensive reviews.

[8] Note that they do not use sectoral productivity.

[9] If I instead targeted changes in wages, my model can closely replicate the changes of sectoral prices and RER (Figure 6), so my quantitative results are, in this sense, complementary to those in Devereux et al. (2025).

tradable good (T) and a non-tradable good (NT) using labor as the only input. Labor is perfectly mobile across sectors within each country, implying a single wage w_i. Production in sector s is characterized by productivity $A_{i,s}$, so unit cost pricing implies

$$P_{i,s} = \frac{w_i}{A_{i,s}}. \tag{1}$$

Under free trade in tradables, the law of one price holds:

$$P_{i,T} = P_{j,T}, \tag{2}$$

which in turn implies that relative wages are pinned down by relative tradable-sector productivity:

$$\frac{w_i}{w_j} = \frac{A_{i,T}}{A_{j,T}}. \tag{3}$$

Given this wage ratio, the relative price of non-tradables between countries i and j is

$$\frac{P_{i,NT}}{P_{j,NT}} = \frac{(w_i/A_{i,NT})}{(w_j/A_{j,NT})} = \frac{(A_{i,T}/A_{j,T})}{(A_{i,NT}/A_{j,NT})}. \tag{4}$$

The aggregate price level in each country is a Cobb–Douglas composite of tradable and non-tradable prices:

$$P_i = P_{i,T}^{\alpha_{i,T}} P_{i,NT}^{\alpha_{i,NT}}, \qquad \text{where } \alpha_{i,T} + \alpha_{i,NT} = 1. \tag{5}$$

The real exchange rate between countries i and j is defined as

$$RER_{i,j} \equiv \frac{P_i}{P_j}. \tag{6}$$

Substituting the expressions above, the real exchange rate becomes

$$RER_{i,j} = \frac{\left(\frac{A_{i,T}}{A_{i,NT}}\right)^{\alpha_{i,NT}}}{\left(\frac{A_{j,T}}{A_{j,NT}}\right)^{\alpha_{j,NT}}}. \tag{7}$$

Hence, a country experiencing faster productivity growth in tradables relative to non-tradables—compared with its trading partner—will experience a real appreciation. This provides the theoretical foundation for my empirical specification testing the Balassa–Samuelson effects.

2.2 Data Sources and Variable Construction

I construct a panel dataset that combines information on real exchange rates and sectoral productivity across countries. The real exchange rate ($RER_{i,t}$) is obtained from the Penn World Table (PWT 11.0) as the ratio of the consumer price index (pl_c) to the nominal exchange rate (xr) relative to the United States (Feenstra et al., 2015). For sectoral productivity, I draw on multiple harmonized production databases. For Europe, the United States, and Japan, I use the *EU KLEMS* dataset (2008 and 2023 releases).[10] For China, I use the *China Industrial Productivity (CIP)* database from RIETI.[11] For Korea, India, and Taiwan, I use *Asia KLEMS*.[12]

Sectoral productivity in each country and year is measured as real value added divided by total hours worked. I refer to this as sectoral average labor productivity (ALP). As a robustness check, I alternatively use a composition-adjusted labor input index; the results remain similar. I classify agriculture, mining, and manufacturing as tradable sectors, while services are treated as non-tradable. For each country and year, I aggregate real value added and labor input across tradable (or non-tradable) industries. Aggregate sectoral productivity is then computed as total real value added divided by total labor input within each group. Country-by-country coverage (start and end years) and data sources are summarized in Table A1 in the Appendix.

To isolate long-run movements consistent with the Balassa–Samuelson mechanism, I remove cyclical components from all productivity and price series using the Hodrick–Prescott filter with a smoothing parameter of $\lambda = 6.25$, following the scaling rule of Ravn and Uhlig (2002). Specifically, I first take logarithms, apply the filter, and then convert the series back to levels. Using alternative filters (e.g., the Christiano–Fitzgerald band-pass filter, Christiano and Fitzgerald (2003)) yields nearly identical results.

[10] EU KLEMS 2008 covers 1970–2005; EU KLEMS 2023 covers 1995–2021. I take 1970–1994 from the 2008 release and 1995–2021 from the 2023 release, and splice at 1995 by multiplicatively normalizing the 1970–1994 series so that the 1995 level matches the 2023 release.

[11] CIP3 covers 1981–2010; CIP4 covers 1987–2017. I take 1981–1986 from CIP3 and 1987–2017 from CIP4, splicing at 1987 by multiplicatively normalizing the 1981–1986 segment to match the 1987 level in CIP4.

[12] Asia KLEMS covers 1980–2012. The data for India do not include hours worked in any period. I instead use value added per worker (composition-adjusted). Excluding India does not change any of the results.

2.3 Traditional Empirical Design and Its Problem

I start from the standard benchmark used in the empirical Balassa–Samuelson literature: I regress the log real exchange rate *relative to a fixed reference country U* on the home tradable–non-tradable productivity differential, controlling for country fixed effects. Formally,

$$\ln RER_{i,t} = \beta \underbrace{\ln\left(\frac{A_{i,T,t}/A_{i,NT,t}}{A_{U,T,t}/A_{U,NT,t}}\right)}_{\text{Log Rel. ALP}} + \mu_i + \varepsilon_{i,t}, \tag{8}$$

where $RER_{i,t} = P_{i,t}/P_{U,t}$ is the bilateral real exchange rate vis-à-vis U, β is the Balassa–Samuelson elasticity, and μ_i are country fixed effects. Equation (8) is the natural stochastic analogue of the two-country model in Section 2.1, where differences in the tradable–non-tradable productivity ratio map into differences in price levels.

Column (1) of Table 1 estimates the model in equation (8) with the United States as the reference over the full sample period, 1970–2021. The estimate is negative, which is inconsistent with the Balassa-Samuelson effect. Column (2) repeats the U.S.-normalized regression for the sample period between 2000 and 2019 (to align with the euro introduction in 1999 and to avoid the COVID period from 2020); the coefficient becomes imprecise, with a negative point estimate. This motivates exploring alternative references—both because BS effects may be clearer when nominal exchange-rate noise is muted (as in the euro area; Berka et al. 2018) and because dollar movements can reflect forces unrelated to productivity (Canzoneri et al., 1999).

Columns (3) and (4) switch the reference to Germany and the United Kingdom, respectively, over 2000–2019. The German benchmark yields a larger, positive coefficient with higher precision.[13] The U.K. benchmark also produces a positive and precisely estimated elasticity.

To see how strongly the choice of reference matters, I repeat the regression in equation (8) separately for each potential benchmark country. This exercise makes explicit how sensitive the estimated Balassa–Samuelson elasticity is to the numeraire adopted in conventional specifications. Earlier studies typically fix the United States as the reference, but there is no theoretical reason why the relationship in Section 2.1 should depend on one country alone. By systematically varying the benchmark, I can assess whether the lack of support in Table 1 reflects genuine model failure or benchmark-specific noise.

Figure 1 summarizes the estimated coefficients from these regressions. Each horizon-

[13] This is broadly consistent with the view that productivity–RER comovement may be easier to detect in settings closer to a common-currency environment and when sectoral wedges are accounted for (Berka et al., 2018; Devereux et al., 2025).

Table 1: Real Exchange Rate and Relative Productivity

	(1)	(2)	(3)	(4)
Log Rel. ALP	-0.15	-0.02	0.40	0.68
	(0.06)	(0.22)	(0.05)	(0.10)
Observations	1,307	631	631	631
Num of Countries	33	33	33	33
Num of Years	52	20	20	20
Sample Years	1970-2021	2000-2019	2000-2019	2000-2019
Ref. Country	U.S.	U.S.	Germany	U.K.

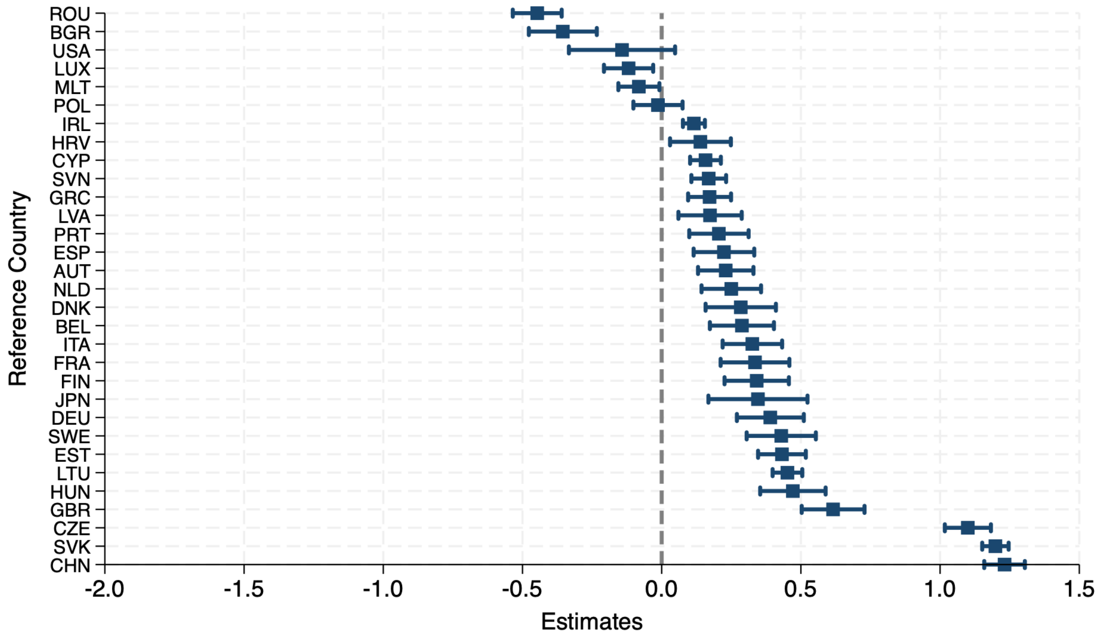
Notes: This table reports panel regressions of the log bilateral real exchange rate against the log relative labor productivity (ALP) differential with the reference country, as specified in equation (8). The dependent variable is $\ln RER_{i,t} = \ln(P_{i,t}/P_{U,t})$, and the explanatory variable is the log difference between tradable and non-tradable productivity in country i relative to reference country U. All regressions include country fixed effects, and standard errors are clustered by country (shown in parentheses). Column (1) uses the United States as the reference over 1970–2021. Column (2) repeats the U.S.-normalized regression for the sample between 2000 and 2019. Column (3) switches the reference to Germany over 2000–2019, and column (4) uses the United Kingdom as an alternative reference country over the same period.

tal line shows the coefficient on the home tradable–non-tradable productivity differential from a separate regression using a different reference country, with 95% confidence intervals clustered by country. The vertical line at zero corresponds to the null of no Balassa–Samuelson effect. Countries are sorted by the magnitude of the estimated elasticity.

The results reveal substantial heterogeneity across reference countries. Several benchmarks yield positive and statistically significant coefficients consistent with the Balassa–Samuelson prediction, including Italy, France, Japan, and Germany (DEU). Nevertheless, the magnitudes vary even among benchmarks with positive estimates. In contrast, four reference countries—Romania, Bulgaria, Luxembourg, and Malta—produce negative and significant estimates. In addition, several benchmarks, including the United States, yield statistically insignificant estimates.

This dispersion suggests a mechanical problem with the single-reference design rather than a small amount of sampling noise. Because every observation is measured relative to U, any shocks or measurement errors specific to the benchmark country are repeated once for each partner and are never averaged out across countries. In the next subsection, I formalize this point in a simple data-generating process that mirrors the classical Balassa–Samuelson model and show that, without time fixed effects, the effective sample size is only T, not NT, so benchmark-specific shocks remain prominent even in large panels.

Figure 1: Balassa–Samuelson Elasticity by Reference Country, 2000-2019



Notes: This figure reports the estimated coefficients from panel regressions of the log bilateral real exchange rate against the log relative labor productivity (ALP) differential with each possible reference country, based on the specification in equation (8). The dependent variable is $\ln RER_{i,t} = \ln(P_{i,t}/P_{U,t})$, and the explanatory variable is the log difference between tradable and non-tradable productivity in country i relative to reference country U. Each horizontal line represents the coefficient and its 95% confidence interval from a separate regression using a different reference country. All regressions include country fixed effects, use data for 2000-2019, and cluster standard errors by country. The vertical line at zero corresponds to the null of no Balassa–Samuelson effect.

2.4 More Robust Specification with Time Fixed Effects

The previous subsection documented that single-reference regressions such as equation (8) deliver coefficients that vary sharply with the chosen benchmark. To understand the source of this instability—and how to address it—I place the traditional regression in a simple statistical environment that is as close as possible to the two-sector Balassa–Samuelson model in Section 2.1.

A simple data-generating process. Let $p_{i,t} \equiv \ln P_{i,t}$ denote the log CPI in country i expressed in a common currency. The stochastic analogue of equation (5) is

$$p_{i,t} = \beta\, a_{i,t} + \alpha_i + g_t + u_{i,t}, \qquad a_{i,t} \equiv \ln\left(\frac{A_{i,T,t}}{A_{i,NT,t}}\right),$$

where β is the Balassa–Samuelson elasticity, α_i collects country-specific constants (including expenditure shares), g_t is a period-common disturbance, and $u_{i,t}$ is an idiosyncratic error.

The empirical object in equation (8) is the bilateral real exchange rate against a reference country U,

$$r_{i,t} \equiv \ln RER_{i,t} = \ln\left(\frac{P_{i,t}}{P_{U,t}}\right) = p_{i,t} - p_{U,t}, \qquad i \neq U.$$

Subtracting the equation for U from that for i yields

$$r_{i,t} = \beta\big(a_{i,t} - a_{U,t}\big) + (\alpha_i - \alpha_U) + \big(u_{i,t} - u_{U,t}\big),$$

because the common component g_t cancels out. Defining

$$x_{i,t} \equiv a_{i,t} - a_{U,t}, \qquad \mu_i \equiv \alpha_i - \alpha_U, \qquad \varepsilon_{i,t} \equiv u_{i,t} - u_{U,t},$$

the population relationship becomes

$$r_{i,t} = \beta x_{i,t} + \mu_i + \varepsilon_{i,t},$$

which is exactly equation (8). The key feature is that the same benchmark error $u_{U,t}$ enters $\varepsilon_{i,t}$ for *every* i in period t: the reference-country shock is replicated across the entire cross-section.

Traditional estimator. I now study the within-country estimator that underlies the single-reference regressions. For notational convenience, I keep U in the sample and note that $r_{U,t} = x_{U,t} = 0$ by construction. Estimating equation (8) with country fixed effects yields the usual within regression

$$\tilde{r}_{i,t} = \beta\,\tilde{x}_{i,t} + \tilde{\varepsilon}_{i,t},$$

where tildes denote deviations from country means:

$$\tilde{x}_{i,t} = (a_{i,t} - \bar{a}_i) - (a_{U,t} - \bar{a}_U), \qquad \tilde{\varepsilon}_{i,t} = (u_{i,t} - \bar{u}_i) - (u_{U,t} - \bar{u}_U).$$

The slope estimator can be written as

$$\hat{\beta} = \beta + \frac{S_{x\varepsilon}}{S_{xx}}, \qquad S_{x\varepsilon} \equiv \sum_{i,t} \tilde{x}_{i,t}\tilde{\varepsilon}_{i,t}, \quad S_{xx} \equiv \sum_{i,t} \tilde{x}_{i,t}^2.$$

To see how the benchmark enters, expand the leading pieces of the score and the quadratic form:

$$
\begin{aligned}
S_{x\varepsilon} &= \sum_{i,t}(a_{i,t}-\bar{a}_i)(u_{i,t}-\bar{u}_i) - \sum_{i,t}(a_{i,t}-\bar{a}_i)(u_{U,t}-\bar{u}_U) \\
&\quad - \sum_{i,t}(a_{U,t}-\bar{a}_U)(u_{i,t}-\bar{u}_i) + N\sum_{t}(a_{U,t}-\bar{a}_U)(u_{U,t}-\bar{u}_U), \\
S_{xx} &= \sum_{i,t}(a_{i,t}-\bar{a}_i)^2 + N\sum_{t}(a_{U,t}-\bar{a}_U)^2 + \text{cross terms.}
\end{aligned}
$$

The last term in each expression comes from the benchmark and is multiplied by N: every additional country brings in one more copy of the same time series for $a_{U,t}$ and $u_{U,t}$.

Bias. Suppose first that strict exogeneity holds for all countries, including the reference,

$$
E[u_{i,t} \mid \{a_{j,\tau}\}_{j,\tau}] = 0 \quad \text{for all } i, t.
$$

Then each summand in $S_{x\varepsilon}$ has zero expectation, so $E[\hat{\beta}] = \beta$. In contrast, if the benchmark violates strict exogeneity so that

$$
\sum_{t}(a_{U,t}-\bar{a}_U)(u_{U,t}-\bar{u}_U) \neq 0,
$$

the last term in $S_{x\varepsilon}$ has nonzero expectation and scales with N. Because the same N-scaled factor appears in S_{xx}, the probability limit of $\hat{\beta}$ shifts to a benchmark-specific value that differs from β. This provides a mechanical interpretation of the wide dispersion across reference countries in Figure 1.

Sampling variation and lack of cross-sectional averaging. Even under strict exogeneity for all countries, including U, the dominant random variation in $S_{x\varepsilon}$ comes from the benchmark terms that repeat across all i. Intuitively, each year's realization of $(a_{U,t}, u_{U,t})$ is used once for every partner country, so cross-sectional averaging cannot reduce its influence; adding more countries simply adds more copies of the same noise. With $N, T \to \infty$,

$$
\hat{\beta} - \beta = \frac{O_p(N\sqrt{T})}{O_p(NT)} + o_p(1) = O_p\!\left(\tfrac{1}{\sqrt{T}}\right),
$$

so the effective sample size is T, not NT. In other words, without time fixed effects, the single-reference estimator does not "wash out" benchmark shocks by enlarging the panel; the uncertainty around $\hat{\beta}$ continues to be governed by the particular time-series history of the reference country, even in very large cross-sections.

Efficiency. Under weak dependence over time,

$$\sqrt{T}\,(\hat{\beta}-\beta) \;\Rightarrow\; \mathcal{N}(0,\sigma^2_{\text{bench}}),$$

so the conventional single-reference estimator converges at the time-series rate $\sqrt{T}$ rather than the panel rate $\sqrt{NT}$. This slow rate is the sampling counterpart of the benchmark-driven fragility seen in Figure 1.

Adding time fixed effects. A minimal way to remove the benchmark-common component is to add year dummies while keeping the single-reference structure. I estimate

$$\ln RER_{i,t} = \beta \ln\left(\frac{A_{i,T,t}/A_{i,NT,t}}{A_{U,T,t}/A_{U,NT,t}}\right) + \mu_i + \tau_t + \varepsilon_{i,t}, \tag{9}$$

where the dependent variable and regressor are exactly the same as in equation (8), but I now include a full set of time fixed effects τ_t. In terms of the DGP above, the time effects absorb any component that is common across countries in a given year, including the repeated benchmark shock $u_{U,t}$.

Let $\ddot{z}_{i,t}$ denote deviations from both country and time means. The within estimator for equation (9) is

$$\ddot{r}_{i,t} = \beta\,\ddot{x}_{i,t} + \ddot{\varepsilon}_{i,t}.$$

Because the double-demeaning removes all period-common terms from both the regressor and the error, the benchmark-specific piece no longer appears in the score. Under the same strict exogeneity condition as above, I have $E[\ddot{x}_{i,t}\ddot{\varepsilon}_{i,t}] = 0$ and hence

$$\operatorname{plim}\hat{\beta} = \beta.$$

Moreover, the remaining sampling variation now accumulates over both i and t. Under weak dependence,

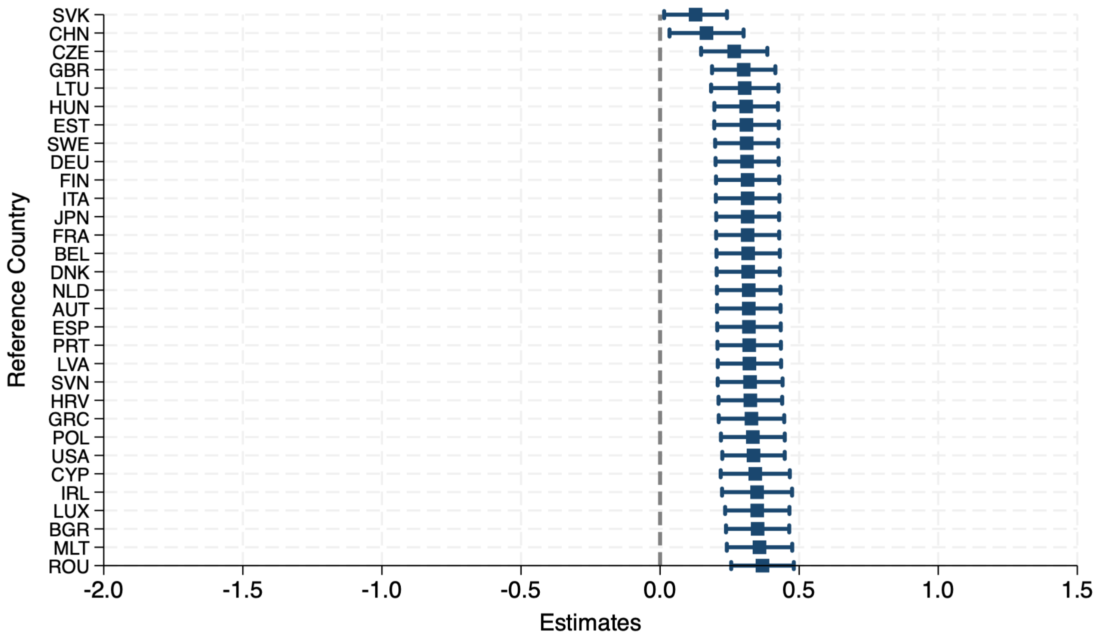
$$\sqrt{NT}\,(\hat{\beta}-\beta) \;\Rightarrow\; \mathcal{N}(0,\sigma^2_{\text{timeFE}}),$$

so the time-FE specification restores the usual panel rate $\sqrt{NT}$ and eliminates the overweighting of the reference country.

Main result. Figure 2 plots the elasticity from equation (9) for each benchmark over 2000–2019. Once time effects are included, the dispersion across reference countries collapses: estimated coefficients are stable in sign and magnitude and no longer hinge on the numeraire. This is exactly what the econometric argument above would predict.

Time fixed effects remove the benchmark-common shocks that previously dominated the single-reference estimator and recover a robust, positive Balassa–Samuelson elasticity.

Figure 2: Balassa–Samuelson Elasticity by Reference Country, 2000-2019, with time fixed effects



Notes: Each line reports the coefficient from a separate panel regression of the log bilateral real exchange rate relative to a given benchmark on the log tradables–nontradables productivity differential relative to that benchmark, as in equation (9). The dependent variable is $\ln RER_{i,t} - \ln RER_{U,t}$. The regressor is $\ln(A_{i,T,t}/A_{i,NT,t}) - \ln(A_{U,T,t}/A_{U,NT,t})$. The sample is 2000-2019. All regressions include country fixed effects and time fixed effects. Standard errors are panel-corrected to allow correlation across countries within a year using the Beck–Katz method with period correlation. Countries are sorted by the point estimate. The vertical line at zero marks the null of no Balassa–Samuelson effect.

2.5 Robustness for the Main Time-FE Specification

Alternative samples. I test whether the main time-FE result depends on sample composition. I first extend the window to 1970–2021 to use all available observations. I then restrict the sample to 2000–2019 to align with the euro introduction in 1999 and to avoid the COVID period from 2020 onward. Next, I focus on advanced economies, where measurement and institutions are more comparable across countries. Finally, I impose a balanced panel from 2000 to 2019 to hold composition fixed over time. Table 2 shows a stable and positive elasticity across all four choices. Magnitudes vary modestly, but the confidence intervals overlap widely, indicating that the time-FE result is not driven by sample selection.

Table 2: Single-reference time-FE regressions: alternative samples

	(1)	(2)	(3)	(4)
Log Rel. ALP	0.10	0.35	0.31	0.35
	(0.04)	(0.06)	(0.06)	(0.06)
Observations	1,307	631	580	560
Sample Countries	All	All	Adv	Balanced
Num of Countries	33	33	29	28
Sample Years	1970-2021	2000-2019	2000-2019	2000-2019
Num of Years	52	20	20	20
Country & Year FE	✓	✓	✓	✓

Notes: This table reports panel regressions based on equation (9) with country fixed effects and time fixed effects. The dependent variable is $\ln RER_{i,t} - \ln RER_{U,t}$ and the regressor is $\ln(A_{i,T,t}/A_{i,NT,t}) - \ln(A_{U,T,t}/A_{U,NT,t})$. Column (1) uses the full 1970–2021 sample. Column (2) restricts to 2000–2019. Column (3) restricts to advanced economies. Column (4) uses a balanced panel for 2000–2019. Standard errors are panel-corrected to allow correlation across countries within a year using the Beck-Katz method with period correlation.

Low-frequency variation. I examine whether the result reflects very slow-moving co-movement. I average the trend components into non-overlapping 5-year, 10-year, and 20-year intervals.[14] I repeat each exercise on the set of countries that are observed in every year between 2000 and 2019. Table 3 shows that the estimated elasticity remains positive and of similar magnitude even when attention is restricted to long-horizon movements, confirming that the finding is neither an artifact of higher-frequency noise nor a pattern that holds only at annual frequencies.

Further robustness. In Appendix B, I report additional robustness checks. Table A2 first separates the original Balassa–Samuelson effect—the relationship between the RER and relative productivity in tradable versus non-tradable sectors—from the Penn effect—the relationship between the RER and economy-wide productivity measures, such as GDP per worker. Second, Table A3 shows that the results are unchanged when I use an alternative dataset, the GGDC 10-Sector Database. Third, Table A4 shows that the results qualitatively hold when I use the labor index measure, which adjusts for compositional changes in the labor force. Finally, Table A5 shows that the results do not depend on whether I use HP-filtered data or unfiltered data.

[14]To avoid the COVID year 2020, I use 2019 instead.

Table 3: Single-reference time-FE regressions: low-frequency variation

	(1)	(2)	(3)	(4)	(5)
Log Rel. ALP	0.30	0.35	0.36	0.31	0.34
	(0.11)	(0.13)	(0.18)	(0.12)	(0.17)
Observations	221	119	74	145	87
Num of Countries	29	29	29	29	29
Frequencies	5 Years	10 Years	20 Years	5 Years	10 Years
Sample Periods	1980-2019	1980-2019	1980-2019	2000-2019	2000-2019
Num of Years	9	5	3	5	3
Country & Year FE	✓	✓	✓	✓	✓

Notes: This table reports panel regressions based on equation (9) with country fixed effects and time fixed effects. Variables are first HP-filtered to extract trends and then aggregated. Columns report 10-year averages, 20-year averages, and long differences where indicated. Balanced-panel variants repeat the same constructions, holding country composition fixed. The dependent variable is $\ln RER_{i,t} - \ln RER_{U,t}$ and the regressor is $\ln(A_{i,T,t}/A_{i,NT,t}) - \ln(A_{U,T,t}/A_{U,NT,t})$. Standard errors are panel-corrected to allow correlation across countries within a year using the Beck-Katz method with period correlation.

3 Model Setup

The empirical results point to a qualitative Balassa–Samuelson (BS) relationship. To quantify its contribution to cross-country price differences, I build a quantitative Armington trade model that features trade frictions and sectoral productivity differences.[15]

3.1 Countries and Sectors

Countries are indexed by $i, j = 1, \dots, N$. Sectors are indexed by $s = 1, \dots, S$. In the quantitative exercises, I focus on $S = 2$, which I label G (goods) and S (services). All prices are expressed in U.S. dollars.

3.2 Preferences

Households in country j have Cobb–Douglas preferences over sectoral composites,

$$U_j = \prod_{s=1}^{S} \left(\frac{C_{j,s}}{\alpha_{j,s}} \right)^{\alpha_{j,s}}, \qquad \sum_{s=1}^{S} \alpha_{j,s} = 1,$$

where $C_{j,s}$ is consumption of sector s in country j, and $\alpha_{j,s}$ are (possibly country-specific) expenditure shares. Let C_j denote aggregate final demand in country j. Optimal allocation

[15] In Appendix D, I show a version with input–output linkages.

implies

$$P_{j,s}C_{j,s} = \alpha_{j,s}P_jC_j, \qquad P_j = \prod_{s=1}^{S} P_{j,s}^{\alpha_{j,s}}.$$

Within each sector s, final demand is a CES composite of varieties produced in different countries,

$$C_{j,s} = \left(\sum_{i=1}^{N} \mu_{i,j,s}^{1/\theta} C_{i,j,s}^{(\theta-1)/\theta} \right)^{\theta/(\theta-1)}, \qquad \theta > 1,$$

where θ is the elasticity of substitution and $\mu_{i,j,s} > 0$ are taste (quality) shifters. Nontradables are obtained as a limiting case: for $s = \text{NT}$, demand is restricted to the domestic variety by setting $\mu_{i,j,\text{NT}} = 1$ if $i = j$ and $\mu_{i,j,\text{NT}} = 0$ otherwise.

The corresponding sectoral price index is

$$P_{j,s} = \left(\sum_{i=1}^{N} \mu_{i,j,s} P_{i,j,s}^{1-\theta} \right)^{1/(1-\theta)},$$

where $P_{i,j,s}$ is the delivered price in country j of the good produced in country i in sector s.

The implied expenditure share of country i in country j's spending on sector s is

$$\pi_{i,j,s} \equiv \frac{P_{i,j,s}C_{i,j,s}}{\sum_{\ell=1}^{N} P_{\ell,j,s}C_{\ell,j,s}} = \frac{\mu_{i,j,s}P_{i,j,s}^{1-\theta}}{\sum_{\ell=1}^{N} \mu_{\ell,j,s}P_{\ell,j,s}^{1-\theta}}.$$

3.3 Technology and Pricing

Labor is the only factor of production. Output in country i and sector s is

$$Y_{i,s} = A_{i,s}L_{i,s},$$

where $A_{i,s}$ is labor productivity and $L_{i,s}$ is labor used in that sector. The unit cost is

$$c_{i,s} = \frac{w_i}{A_{i,s}},$$

where w_i is the wage in country i.

3.4 Trade Costs

Iceberg trade costs $\tau_{i,j,s} \geq 1$ apply to shipments from country i to country j in sector s. Delivering one unit in j requires shipping $\tau_{i,j,s}$ units from i. The delivered price is

$$P_{i,j,s} = \tau_{i,j,s} c_{i,s} = \frac{w_i \tau_{i,j,s}}{A_{i,s}}.$$

Frictionless trade corresponds to $\tau_{i,j,s} \equiv 1$. Non-tradables are captured by shutting down imports in the non-tradable sector, i.e., by setting $\mu_{i,j,\mathrm{NT}} = 1$ if $i = j$ and $\mu_{i,j,\mathrm{NT}} = 0$ otherwise.

3.5 Labor and Income

Total labor supply in country i is $L_i = \sum_s L_{i,s}$ and is taken as exogenous. Labor is perfectly mobile across sectors within a country, so a single wage w_i clears all sectoral labor markets in country i.

I allow for exogenous trade imbalances. Let β_j denote country j's trade surplus as a fraction of world income, scaled by

$$Y \equiv \sum_{\ell=1}^{N} w_\ell L_\ell.$$

By construction, the surpluses sum to zero,

$$\sum_{\ell=1}^{N} \beta_\ell = 0,$$

so that world absorption equals world income. Nominal expenditure (absorption) in country j is

$$P_j C_j = w_j L_j - \beta_j Y.$$

A country with $\beta_j > 0$ runs a surplus and spends less than its income; a country with $\beta_j < 0$ runs a deficit and spends more than its income. When I normalize world income to one, $Y = 1$, this becomes

$$P_j C_j = w_j L_j - \beta_j.$$

3.6 Equilibrium

An equilibrium is a set $\{w_i, P_{j,s}, \pi_{i,j,s}\}_{i,j,s}$ such that goods markets clear, expenditure shares are consistent with CES demand, and the trade balance is consistent with the exogenous trade surpluses $\{\beta_i\}$.

Let

$$Y \equiv \sum_{\ell=1}^{N} w_\ell L_\ell$$

denote aggregate world income. Since β_i is defined as the trade surplus of country i as a share of Y, $\beta_i > 0$ means that country i exports more than it absorbs, and $\sum_i \beta_i = 0$.

Goods-market clearing for exporter i requires

$$\sum_{j=1}^{N} \sum_{s=1}^{S} \pi_{i,j,s} \alpha_{j,s} w_j L_j = w_i L_i + \beta_i Y. \tag{10}$$

The left-hand side is total revenue that country i earns from selling to all destinations and sectors. The right-hand side is factor income $w_i L_i$ plus the exogenous trade surplus $\beta_i Y$. Because the surpluses sum to zero, summing (10) over i yields $Y = Y$.

Bilateral expenditure shares are given by CES demand and delivered prices:

$$\pi_{i,j,s} = \frac{\mu_{i,j,s} \left(\frac{w_i \tau_{i,j,s}}{A_{i,s}} \right)^{1-\theta}}{\sum_{\ell=1}^{N} \mu_{\ell,j,s} \left(\frac{w_\ell \tau_{\ell,j,s}}{A_{\ell,s}} \right)^{1-\theta}}. \tag{11}$$

Given $\{\pi_{i,j,s}\}$ from (11), solving (10) for $\{w_i\}$ delivers sectoral price indices $\{P_{j,s}\}$ and, in turn, the consumer price index,

$$P_j = \prod_{s=1}^{S} P_{j,s}^{\alpha_{j,s}}.$$

The CPI-based bilateral real exchange rate between countries i and j is

$$RER_{i,j} = \frac{P_i}{P_j},$$

which is my target.

4 Quantitative Importance of the Balassa–Samuelson Force

This section simulates the model in a sequence of experiments that increase in realism and data content. I first study a traditional 2×2 free-trade economy (goods sector as tradables T and service sector as non-tradables NT; two countries) to isolate the Balassa–Samuelson mechanism in its simplest form. I then introduce iceberg trade costs in the same 2×2 environment to quantify how costly trade attenuates the transmission from sectoral productivity to prices and the real exchange rate. Next, I move to the $N \times 2$ model disciplined by data for a large set of countries, and—when presenting results—I also report an $N \times 2$ specification with input–output linkages. The IO extension is used for quantitative exercises in the main text, while its structure and solution are detailed in Appendix D. Finally, I work with the $N \times S$ (multi-country, multi-sector) model, where I do not take a stance on the categorization of the multiple sectors into a two-sector dichotomy. In each case, I solve the static equilibrium year by year, recover sectoral price indices and the CPI, and construct $REER_{i,t}$.

I keep the baseline structure deliberately minimal—one factor, Cobb–Douglas across sectors, and Armington within sectors—so that the quantitative contribution of sectoral productivity $\{A_{i,s,t}\}$ and iceberg costs $\{\tau_{i,j,s,t}\}$ to real exchange rates is transparent and directly comparable to the reduced-form evidence. When I employ the IO extension in the $N \times 2$ environment, I maintain the same calibration strategy; details are in Appendix D.

I calibrate the model to 2000 and conduct counterfactual simulations using exact hat algebra, with $\hat{x} \equiv x'/x$. From the data in 2000, I construct share parameters $\{\alpha_{j,s}\}$ and $\{\mu_{i,j,s}\}$. I set $\theta = 5$. The shocks are sectoral productivity $\{\widehat{A}_{i,s}\}$ and iceberg trade costs $\{\widehat{\tau}_{i,j,s}\}$. See Appendix C for details.[16]

4.1 Shock Feeds

Productivity. Sectoral labor productivity $\{\widehat{A}_{i,s,t}\}$ is taken from the same sources as in the empirical section: the EU KLEMS database.[17] I compute sectoral $A_{i,s,t}$ as real value added divided by labor input, aggregate industries into T (agriculture, mining, manufacturing) and NT (services), and apply the Hodrick–Prescott filter with $\lambda = 6.25$ to the log series to

[16]Note that this method of hat algebra can be problematic as it requires the model to fit the data in 2000 exactly. See Antrás and Chor (2022) and Dingel and Tintelnot (2025), which clarify this overfitting issue

[17]To evaluate the model performance in detail, I use sectoral CPI data. The sectoral CPI data are only available for countries in Europe, the US, and Japan, and thus, I drop other Asian countries from the main quantitative exercises. Nevertheless, the implication for REER is the same if I include these countries.

isolate low-frequency movements.[18]

When the model includes input–output linkages, I use an *intermediate-adjusted productivity* that accounts for the role of intermediate inputs in production. Specifically, using cost shares from KLEMS, I back out effective productivity as

$$\ln A^{IO}_{i,s,t} = \ln \text{Gross Output}_{i,s,t} - \gamma_{i,s} \ln L_{i,s,t} - (1 - \gamma_{i,s}) \ln \text{Intermediate}_{i,s,t},$$

where $\gamma_{i,s}$ is the labor cost share. This adjustment ensures that productivity growth reflects efficiency gains net of changes in intermediate input prices, consistent with the IO model's unit-cost structure. I construct $\widehat{A}^{IO}_{i,s,t}$ as the ratio of the filtered series between t and 2000.

Trade Costs. Bilateral iceberg trade costs $\{\widehat{\tau}_{i,j,s,t}\}$ for tradables are inferred from trade data using the OECD Inter-Country Input–Output (ICIO) database (2025 Extended Edition). The ICIO provides bilateral trade flows by sector. I classify agriculture, mining, and manufacturing as goods sectors, and all remaining sectors as services.

In the model with input–output linkages, I separately calibrate iceberg costs for *final* and *intermediate* goods, denoted $\tau^F_{i,j,s,t}$ and $\tau^X_{i,j,s,t}$, respectively. Both are recovered from bilateral trade flows using the standard gravity-based inversion method. In counterfactual simulations, I allow both $\widehat{\tau}^F_{i,j,s,t}$ and $\widehat{\tau}^X_{i,j,s,t}$ to evolve along their estimated low-frequency paths, so that trade frictions adjust consistently across final and intermediate markets.

Following Head and Ries (2001), I back out $\{\tau_{i,j,s,t}\}$ using

$$(\tau_{ijst})^{1-\theta} = \sqrt{\frac{X_{ijst} X_{jist}}{X_{iist} X_{jjst}}},$$

where X_{ijst} is a gross trade flow from i to j in sector s in year t.

As with productivity, I work with the low-frequency component of $\ln \tau_{i,j,T,t}$ obtained by applying the HP filter with $\lambda = 6.25$. I then normalize the level relative to 2000 and obtain $\widehat{\tau}_{i,j,s}$.

Normalization and Timing. I simulate the model year by year over 2000–2019. For countries with missing data, I log-linearly extrapolate the series to obtain a balanced panel. All nominal variables are expressed in U.S. dollars; the model determines wages $\{w_{i,t}\}$ and price indices $\{P_{i,s,t}\}$ up to a common scalar each year. I remove this indeter-

[18] Categorizing IT sectors into tradable sectors does not change results. Also, in Section E.3, I show the results based on 23 sector models. The key message is the same.

minacy by fixing world GDP to one. Country-specific expenditure weights $\{\alpha_{i,s}\}$ are set to average expenditure shares in the data. The trade elasticity is set to $\theta = 5.0$, which is standard.

4.2 Results

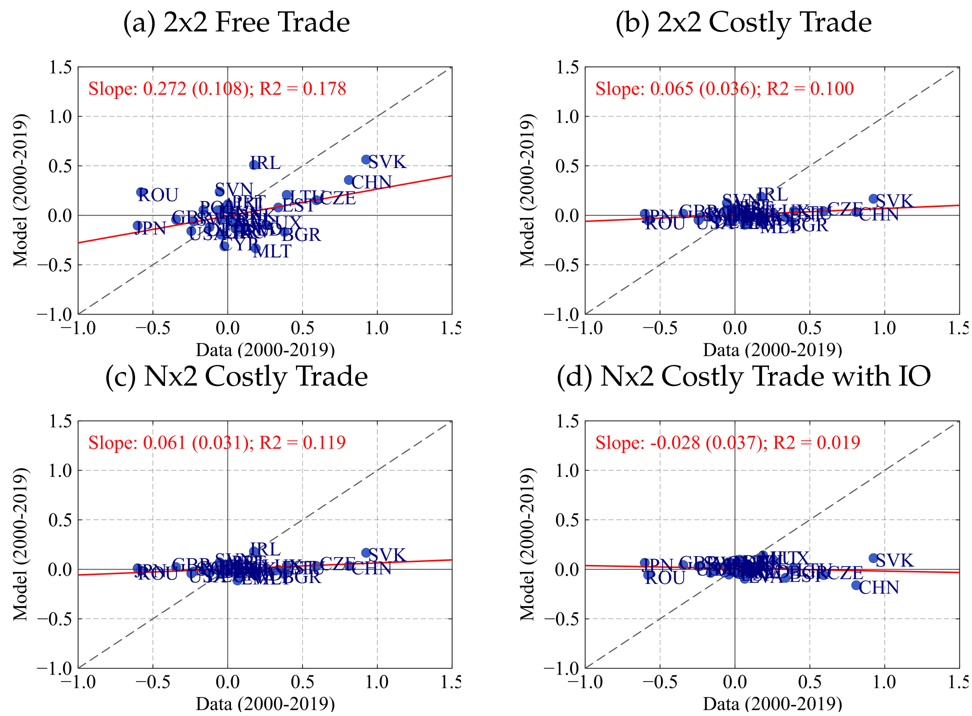
Figure 3 summarizes my first counterfactual. I feed only sectoral productivity shocks—that is, I allow $\{A_{i,s,t}\}$ to evolve as in the data, hold trade costs $\{\tau_{i,j,s,t}\}$ fixed at their 2000 values, and solve the model year by year. I then compare, for each country, the long difference in its real effective exchange rate (REER) between 2000 and 2019 in the data with the corresponding long difference implied by the model. Each panel of Figure 3 reports this comparison under a different version of the model: a 2×2 free-trade case, a 2×2 case with iceberg trade costs, an $N \times 2$ costly-trade case that allows for third-country reallocation, and an $N \times 2$ version that also incorporates input–output linkages.[19] Points on the 45-degree line would indicate that the model with productivity shocks alone reproduces the observed REER movement.

The scatter plots in Figure 3 show that productivity shocks alone cannot replicate observed REER changes. Large appreciations and depreciations in the data translate into much smaller movements in the model, even in the richer $N \times 2$ environments. In many cases, the sign is already off: some countries that appreciate in the data appear as mild depreciations in the model, and vice versa. In other cases, the sign is correct, but the slope is far too flat—the model's long-run REER movement is only a fraction of what is observed.

To examine the time dimension behind these long differences, Appendix Figure A1 plots the full REER paths for six large economies—China, Germany, France, Italy, Japan, and the United States—under the productivity-shock experiment. In each panel, I show the HP-filtered REER from the data (normalized to one in 2000) and the model-implied REER paths generated by feeding only $\{A_{i,s,t}\}$. The qualitative failure is clearest in Japan: the data exhibit a depreciation of more than 0.7 log points between 2000 and 2019, while the model predicts an appreciation. China moves the wrong way as well: the REER in the data appreciates sharply, whereas the model delivers a slight depreciation. Some countries move in the right direction qualitatively—for example, Germany's observed appreciation is mirrored by an appreciation in the model—but even there, the amplitude is far too small. Overall, feeding observed sectoral productivity—the traditional Balassa–Samuelson channel—rarely gets the sign right and is far too weak quantitatively.

[19] See Appendix D for model details.

Figure 3: REER Long-Difference Fit vs. Data, 2000–2019: Productivity Shock Only



Notes: Each dot represents one country. The horizontal axis is the HP-filtered change in log REER between 2000 and 2019 in the data (BIS). The vertical axis is the corresponding change implied by the model when only sectoral productivity $\{A_{i,s,t}\}$ is allowed to move, while iceberg trade costs $\{\tau_{i,j,s,t}\}$ are fixed at 2000 levels. Panels differ by model environment. The solid line is the OLS fit; the slope (standard error) and R^2 are reported in each panel. The 45-degree line indicates a perfect quantitative match.

4.3 Robustness

Adding Trade-Cost Shocks. Figure A2 repeats the long-difference exercise after adding trade-cost shocks. Bilateral iceberg trade costs $\{\tau_{i,j,s,t}\}$ evolve along their estimated low-frequency paths. I solve the model year by year, compute sectoral price indices and the CPI, and compare each country's change between 2000 and 2019 in the data with the corresponding model-implied change. I report three environments: 2×2 with costly trade, $N \times 2$ with costly trade, and $N \times 2$ with costly trade plus input–output linkages.

Results are similar to Figure 3. Allowing time variation in $\{\tau_{i,j,s,t}\}$ delivers only minor improvements, and the quantitative fit remains weak. Large appreciations and depreciations in the data translate into muted changes in the model-implied series. The overall conclusion is unchanged: adding trade-cost reductions does not account for the observed scale of real effective exchange rate movements.

Beyond Two-sector Models. Figure A3 repeats the same exercises in an $N \times S$ setting. Here, I use 23 sectors, including 13 goods sectors and 10 service sectors. The motivation is not only to move beyond a coarse goods–services dichotomy, but also to allow for heterogeneity within goods and within services. This matters because countries can differ sharply in which goods (and which services) they specialize in, and a two-sector aggregation implicitly forces those heterogeneous composition effects to be ignored.

Results are similar to Figure 3. Even with richer sectoral detail, the model continues to generate too little cross-country variation in RER changes relative to the data.

4.4 Productivity and Sectoral Prices

The previous subsection shows that the Balassa–Samuelson mechanism cannot replicate the magnitude—and, in some cases, even the sign—of international price movements in the aggregate CPI. Prior work reports that the Balassa–Samuelson relationship is not evident for aggregate real exchange rates but appears qualitatively in sectoral prices, especially for services (Engel, 1999; Canzoneri et al., 1999), and in some cases for goods (Lee and Tang, 2007). I examine whether observed sectoral productivity changes can quantitatively match international differences in sectoral producer price indices (PPI), in terms of sectoral RER.[20]

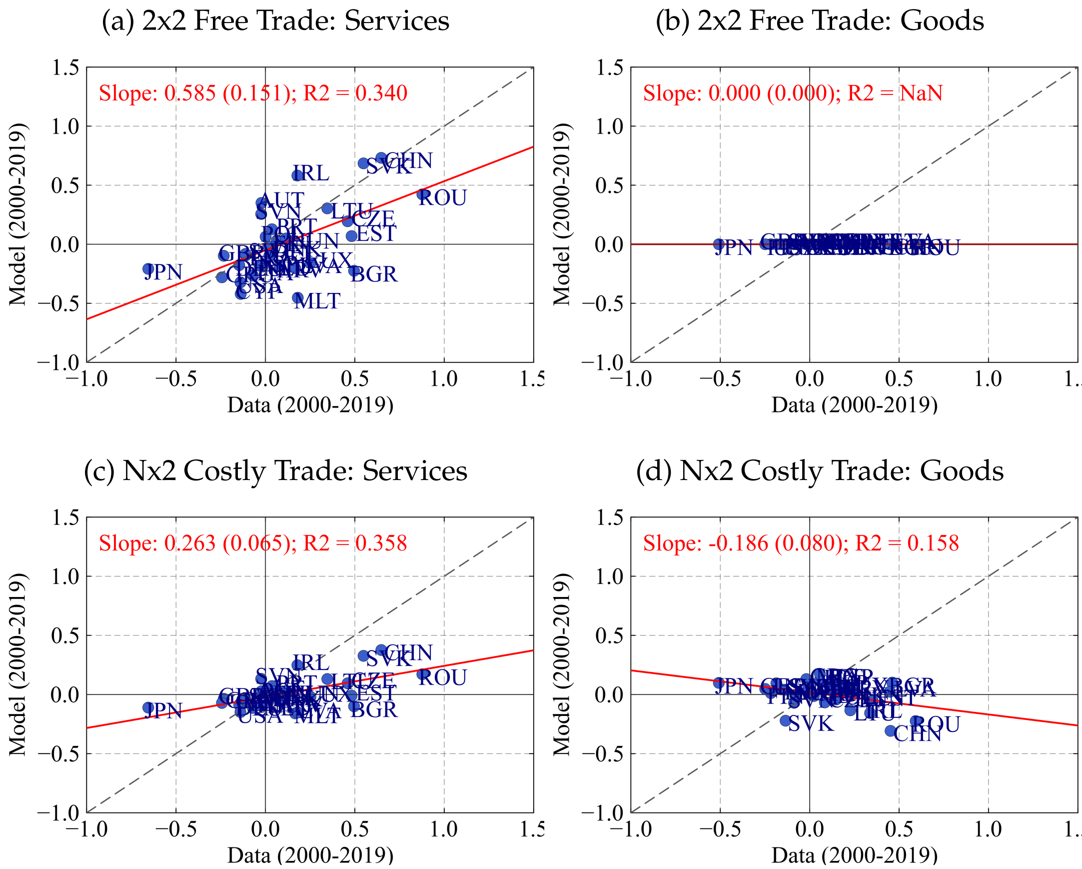
Sectoral Price Fits Figure 4 presents the results. Panels (a) and (b) plot simulated changes in sectoral RERs between 2000 and 2019 against their observed counterparts, where the simulations feed sectoral productivity shocks into a 2×2 free-trade environment. In panel (a), service-sector RERs generally move in the direction implied by relative sectoral productivity and, qualitatively, fit better than the aggregate RER. Nonetheless, the fit remains limited ($R^2 = 0.34$), and the implied magnitudes are still too small. Panel (b) reports results for the goods sector. By construction, free trade equalizes tradable-goods prices across countries, so the model cannot generate the observed cross-country heterogeneity in changes in goods prices.

Panels (c) and (d) switch to an $N \times 2$ environment with costly trade. Service-sector RERs continue to move in the expected direction, and the relationship is somewhat tighter, but the predicted magnitudes remain quantitatively small. For goods, however, the model now predicts movements in the opposite direction. This inability to match goods-sector RER changes carries over to the aggregate RER. In this sense, moving from the textbook

[20] Ideally, I would like to use sectoral CPI, which can be easily aggregated to RER. However, cross-country data for sectoral CPI are surprisingly scarce. Thus, I rely on the EU KLEMS data used for productivity to back out sectoral PPI.

2 × 2 free-trade setup to the $N \times 2$ costly-trade framework does not improve fit in this exercise and may even worsen it.

Figure 4: Sectoral RER: Model Fit vs. Data, Long Differences 2000–2019



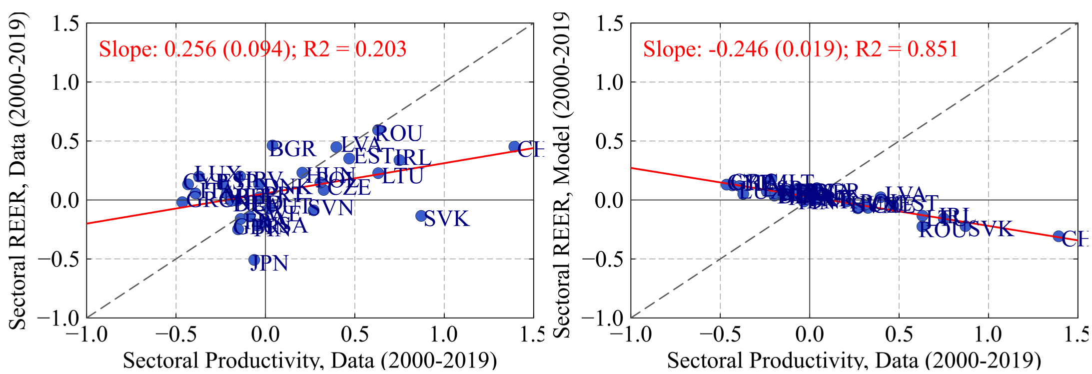
Notes: Each dot represents one country. The horizontal axis is the observed change in the sectoral REER between 2000 and 2019, constructed from sectoral PPIs. The vertical axis shows the corresponding model-implied change when sectoral productivities $\{A_{i,s,t}\}$ evolve along their estimated low-frequency paths. Country-level sectoral PPIs are converted to effective terms using BIS double weighting, following the multilateral index construction in Klau and Fung (2006). The solid line is the OLS fit; the slope (standard error) and R^2 are reported in each panel. A 45-degree line would indicate a perfect quantitative match.

Sectoral Productivity and Sectoral Prices Figure 5 focuses on the goods sector to highlight the sign reversal between the data and the model. Panel (a) plots, for each country, the change in goods-sector productivity (horizontal axis) against the change in the goods-sector RER constructed from sectoral PPIs (vertical axis) between 2000 and 2019. The relationship is positive: the estimated slope is 0.256 (s.e. 0.094) with $R^2 = 0.203$, indicating that countries with faster productivity growth in the goods sector tend to experience larger increases in goods-sector relative prices. For instance, China experiences an increase in goods-sector productivity of 1.5 log points between 2000 and 2019 and an increase in the goods-sector price (in RER) of 0.5 points over the same period.

Panel (b) keeps the same productivity changes on the horizontal axis but replaces the vertical axis with the model-implied change in the goods-sector RER in the $N \times 2$ costly-trade simulation. The relationship flips sign and becomes much tighter: the slope is -0.246 (s.e. 0.019) with $R^2 = 0.851$. In the model, higher goods-sector productivity growth is associated with larger declines in goods-sector relative prices, in contrast to the pattern in the data. Chinese goods prices are expected to decline by about 0.5 log points, instead of increasing by 0.5 points as observed in the data.

Figure 5: Goods Sector Productivity and RER: $N \times 2$ Costly Trade Case

(a) Data (b) Model



Notes: Each dot represents one country. The horizontal axis reports the observed change in goods-sector productivity between 2000 and 2019. The vertical axis reports the change in the goods-sector real exchange rate (sectoral REER) over the same period: panel (a) uses the data measure constructed from sectoral PPIs, while panel (b) uses the model-implied counterpart from the $N \times 2$ costly-trade simulation in which sectoral productivities $\{A_{i,s,t}\}$ follow their estimated low-frequency paths. Country-level sectoral PPIs are converted to effective multilateral prices using BIS double-weighting, following Klau and Fung (2006). The solid line is the OLS fit; the slope (standard error) and R^2 are reported in each panel.

4.5 Source of Quantitative Failures: Wages

So far, the main quantitative failures show up as too little variation in model-implied RER movements, even in the $N \times 2$ environment with time-varying iceberg trade costs, as shown in Figures 3–A2. A natural suspect is wages. In my setup, a single wage $w_{i,t}$ clears all sectoral labor markets in country i and enters every delivered price through unit costs. Because sectoral CES price indices and the Cobb-Douglas CPI are built from unit costs, wages are the equilibrium objects that ultimately determine relative price levels and the RER.

This motivates a diagnostic exercise: taking wages as given. Conceptually, the overall failure could come from the wage block (the general-equilibrium mapping from $\{A_{i,s,t}, \tau_{i,j,s,t}\}$ into $w_{i,t}$) or from the pricing block (the CES/Armington structure translating wages into sectoral and aggregate prices). By feeding wages directly, I bypass the first margin and ask whether the model would match RER movements if it were handed the correct wage path. This experiment should be interpreted with caution, because wages are endogenous, and feeding them directly amounts to injecting the reduced-form effects of omitted shocks and wedges into the model.

Figure 6 implements this idea. I replace the model-implied wage series with observed nominal wages (in national currency) and then compute the model-implied change in sectoral RERs and the overall RER using the same CES aggregators and trade structure, with feeding sectoral productivity paths as in the data. The horizontal axis reports the HP-filtered long difference in log RER in the data, and the vertical axis reports the corresponding long difference implied by the model when wages are taken as given.

Panels (a) and (b) report the sectoral RERs. For services, the model lines up closely with the data (slope 0.876 with s.e. 0.058, $R^2 = 0.886$). For goods, the relationship is still positive but weaker and more attenuated (slope 0.532 with s.e. 0.104, $R^2 = 0.475$).

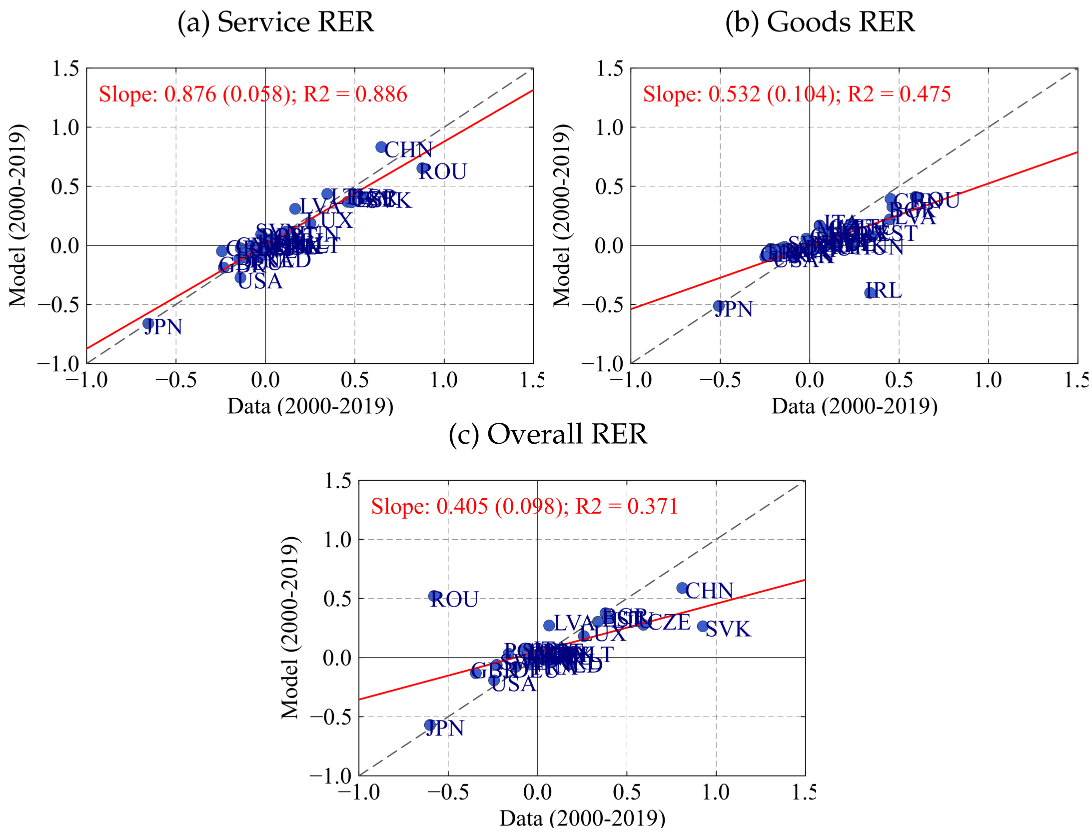
Panel (c) shows the overall RER. The model captures part of the cross-country variation (slope 0.405 with s.e. 0.098, $R^2 = 0.371$), but the fit is noticeably weaker than for services, and a few outliers—most visibly Romania and Slovakia—remain.

Taken together, Figure 6 suggests that explaining wages is a key missing ingredient for a successful quantitative account of long-run RER dynamics in this framework. Understanding why real wages diverge so much across countries—beyond measured sectoral productivity and iceberg trade costs—is therefore central, and I am actively exploring mechanisms that can generate the missing wage movements.

4.6 Discussion: Perfect Competition and TFP Measurement.

My baseline maintains perfect competition. A natural concern is that the model's difficulty matching long-run RER paths could instead reflect imperfect competition in product or factor markets (markups or labor markdowns), suggesting that one might introduce wedges and replace sectoral productivities with "markup-adjusted TFP." The measurement problem is that EU KLEMS/INTANProd growth accounting constructs capital compensation using an *internal* rate of return that exhausts non-labor income, so revenue shares proxy output elasticities only under perfect competition; with markups or markdowns, revenue-based residuals mix technology with wedges and non-CRS forces, and a

Figure 6: Prices by Feeding Wages, Long Differences 2000–2019



Notes: Each dot represents one country. The horizontal axis reports the HP-filtered long difference (2000–2019) in the log RER in the data: panels (a) and (b) use sectoral RERs constructed from sectoral PPIs and converted to effective multilateral indices using BIS double-weighting (as in Klau and Fung (2006)); panel (c) uses the BIS real effective exchange rate. The vertical axis reports the corresponding long difference implied by the $N \times 2$ costly-trade model when the equilibrium wage sequence is replaced by observed nominal wages in national currencies, holding fixed the rest of the model structure and feeding sectoral productivity paths as in the data. The solid red line is the OLS fit; the slope (standard error) and R^2 are reported in each panel. The dashed 45-degree line indicates a perfect match.

markup-consistent A is not identified from KLEMS alone (Bontadini et al., 2023).[21] Recent aggregation methods show how to incorporate market power using firm-level markups and imputed input shares for the United States (Baqaee and Farhi, 2020), but extending this approach country-by-country would require comparable firm-level coverage and harmonized capital-cost imputations that are not available across my panel. Therefore, I retain perfect competition as my maintained assumption and interpret sectoral A from KLEMS accordingly.

5 Conclusion

This paper places the Balassa–Samuelson hypothesis on an empirically cleaner footing and evaluates its quantitative relevance. In the data, adding time fixed effects yields the expected signs and produces a stable, positive elasticity, overturning the mixed and sample-dependent results generated by the specification commonly used in the literature. However, when I translate these elasticities into implied country-level paths, the quantitative impact is small. The shortfall is most striking for Japan: a 0.7-log depreciation in the data between 2000 and 2019 is essentially invisible to the standard sectoral-productivity Balassa–Samuelson effect.

A standard multi-country Armington model reinforces this message. Feeding observed productivity growth does not replicate the paths of real exchange rates. Matching both the level and slope of RERs likely requires additional ingredients—for example, quality shifts within sectors (Fieler, 2011), wedges in labor markets (Devereux et al., 2025), demand shifts (Bergstrand, 1991), factor intensity (Bhagwati, 1984), sectoral markups, demographics, or more realistic terms-of-trade movements. These remain promising directions for future research.

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[21] See Takahashi and Takayama (2025a,b) for a detailed discussion of this point.

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Appendix

Contents

A Additional Data Details

A.1 Coverage of the Data

Table A1: Coverage in Empirical Analyses

Country	Start year	End year	Data source
AUT	1970	2021	EU KLEMS
BEL	1970	2021	EU KLEMS
BGR	1995	2021	EU KLEMS
CHN	1981	2017	CIP
CYP	1995	2021	EU KLEMS
CZE	1995	2021	EU KLEMS
DEU	1970	2021	EU KLEMS
DNK	1970	2021	EU KLEMS
ESP	1970	2021	EU KLEMS
EST	1995	2021	EU KLEMS
FIN	1970	2021	EU KLEMS
FRA	1970	2021	EU KLEMS
GBR	1970	2021	EU KLEMS
GRC	1970	2021	EU KLEMS
HRV	1995	2021	EU KLEMS
HUN	1992	2021	EU KLEMS
IND	1981	2009	Asia KLEMS
IRL	1970	2021	EU KLEMS
ITA	1970	2021	EU KLEMS
JPN	1973	2021	EU KLEMS
KOR	1980	2012	Asia KLEMS
LTU	1995	2021	EU KLEMS
LUX	1970	2021	EU KLEMS
LVA	1995	2021	EU KLEMS
MLT	2000	2021	EU KLEMS
NLD	1970	2021	EU KLEMS
POL	1995	2021	EU KLEMS
PRT	1970	2021	EU KLEMS
ROU	1995	2021	EU KLEMS
SVK	1995	2021	EU KLEMS
SVN	1995	2021	EU KLEMS
SWE	1970	2021	EU KLEMS
TWN	1980	2009	Asia KLEMS

B More Robustness for Empirics

B.1 Penn effect.

I ask whether the single-reference result is absorbed by aggregate productivity. I augment the regression with the relative GDP per worker from the Penn World Table and re-estimate on the 2000-2019 period as well as the extended 1970–2021 window. Table A2 shows that the Balassa–Samuelson coefficient remains positive and precisely estimated after adding the Penn effect control. The Penn coefficient weakens and can change sign when extending the window back to 1970, whereas the BS term remains stable.

Table A2: Single-reference time-FE regressions with Penn effect control

	(1)	(2)	(3)	(4)
Log Rel. ALP	0.34		0.25	0.25
	(0.06)		(0.07)	(0.05)
Log Rel. GDP per Workers		0.27	0.14	-0.33
		(0.06)	(0.06)	(0.11)
Observations	689	689	689	1,307
Sample Countries	All	All	All	All
Num of Countries	33	33	33	33
Sample Years	2000-2019	2000-2019	2000-2019	1970-2021
Num of Years	22	22	22	52
Country & Year FE	✓	✓	✓	✓

Notes: This table reports panel regressions based on equation (9) augmented with $\ln(\text{GDPpw}_{i,t}) - \ln(\text{GDPpw}_{U,t})$ from the Penn World Table. All specifications include country fixed effects and time fixed effects. Columns report estimates for 2000-2019 and for 1970–2021, as indicated in the table body. The dependent variable is $\ln RER_{i,t} - \ln RER_{U,t}$. The BS regressor is $\ln(A_{i,T,t}/A_{i,NT,t}) - \ln(A_{U,T,t}/A_{U,NT,t})$. Standard errors are panel-corrected to allow correlation across countries within a year using the Beck–Katz method with period correlation.

B.2 Different data.

I test portability to a broader setting. I switch to the GGDC 10-Sector Database which extends coverage to a larger set of economies and a longer period. I estimate the same time-FE specification with and without the Penn effect control on the full 1960–2013 span and on the post-2000 subset. Table A3 indicates that the elasticity remains positive across both samples. The magnitude is smaller in the late period but the sign and significance persist, showing that the time-FE design is robust to alternative sources and wider coverage.

Table A3: Single-reference time-FE regressions: GGDC 10-sector data

	(1)	(2)	(3)	(4)
Log Rel. ALP	0.84	0.43	0.65	0.35
	(0.07)	(0.07)	(0.07)	(0.07)
Log Rel. GDP per Workers			1.42	-0.33
			(0.13)	(0.06)
Observations	2,106	546	2,106	546
Sample Countries	All	All	All	All
Num of Countries	39	39	39	39
Sample Years	1960-2013	2000-2013	1960-2013	2000-2013
Num of Years	54	14	54	14
Country & Year FE	✓	✓	✓	✓

Notes: This table reports panel regressions based on equation (9) estimated on the GGDC 10-Sector Database. All specifications include country fixed effects and time fixed effects. Columns report results for 1960–2013 and for 2000–2013, each without and with the Penn effect control as indicated in the table body. The dependent variable is $\ln RER_{i,t} - \ln RER_{U,t}$. The BS regressor is $\ln(A_{i,T,t}/A_{i,NT,t}) - \ln(A_{U,T,t}/A_{U,NT,t})$. Standard errors are panel-corrected to allow correlation across countries within a year using the Beck–Katz method with period correlation.

B.3 Labor composition adjustment.

I ask whether changing the labor input measure affects the time-FE result. I replace hours with the KLEMS composition-adjusted labor index that tracks changes in worker composition over time. I re-estimate the same single-reference time-FE specification on the standard sample set so that the only change is the labor input definition. Table A4 shows that the elasticity remains positive and similar in magnitude across columns. This indicates that the result is not driven by how labor input is measured.

B.4 No time-series filtering.

I check that the time-FE result is not an artifact of trend extraction. I re-estimate the specification on unfiltered annual series while keeping the same country and time fixed effects and the same samples. Table A5 shows that the coefficient remains positive and close to the baseline magnitudes, though standard errors widen as expected when short-run noise is present.

Table A4: Single-reference time-FE regressions: labor composition adjusted

	(1)	(2)	(3)	(4)
Log Rel. ALP	0.78	0.74	0.70	1.06
	(0.07)	(0.07)	(0.07)	(0.06)
Observations	511	460	420	280
Sample Countries	All	Adv.	Europe	Balanced
Num of Countries	33	29	27	14
Sample Years	2000-2019	2000-2019	2000-2019	2000-2019
Num of Years	20	20	20	20
Country & Year FE	✓	✓	✓	✓

Notes: This table reports panel regressions based on equation (9) where labor productivity uses the KLEMS composition-adjusted labor index in place of hours. All specifications include country fixed effects and time fixed effects. All columns cover 2000-2019. Column (1) covers all countries. Column (2) restricts to advanced economies. Column (3) restricts to European countries. Column (4) covers only a balanced panel. The dependent variable is $\ln RER_{i,t} - \ln RER_{U,t}$. The Balassa–Samuelson regressor is $\ln(A_{i,T,t}/A_{i,NT,t}) - \ln(A_{U,T,t}/A_{U,NT,t})$. Standard errors are panel-corrected to allow correlation across countries within a year using the Beck–Katz method with period correlation.

Table A5: Single-reference time-FE regressions: unfiltered annual series

	(1)	(2)	(3)	(4)
Log Rel. ALP	0.08	0.27	0.26	0.26
	(0.04)	(0.07)	(0.06)	(0.06)
Observations	1,307	631	580	580
Sample Countries	All	All	Adv	Balanced
Num of Countries	33	33	29	29
Sample Years	1970-2021	2000-2019	2000-2019	2000-2019
Num of Years	52	20	20	20
Country & Year FE	✓	✓	✓	✓

Notes: This table reports panel regressions based on equation (9) estimated on unfiltered annual data. All specifications include country fixed effects and time fixed effects. Column (1) covers 1970–2021. Column (2) restricts to 2000-2019. Column (3) restricts to advanced economies. Column (4) is a balanced panel for 2000-2019. The dependent variable is $\ln RER_{i,t} - \ln RER_{U,t}$. The Balassa–Samuelson regressor is $\ln(A_{i,T,t}/A_{i,NT,t}) - \ln(A_{U,T,t}/A_{U,NT,t})$. Standard errors are panel-corrected to allow correlation across countries within a year using the Beck–Katz method with period correlation.

C Hat Algebra

Hats denote proportional changes relative to the baseline, e.g. $\hat{x} = x'/x$.

I allow shocks to productivity, trade costs, and labor supply, so the objects $\widehat{A}_{i,s}$, $\widehat{\tau}_{i,j,s}$, and $\hat{L}_i$ can all differ from one. Baseline world income is normalized to one,

$$\sum_{\ell=1}^{N} w_\ell L_\ell = 1,$$

and I keep each country's trade surplus β_i fixed across counterfactuals, with

$$\sum_{\ell=1}^{N} \beta_\ell = 0,$$

where $\beta_i > 0$ means that country i runs a trade *surplus* in the baseline.

Trade balance and labor-market clearing. With a labor-supply shock, post-shock income of country i is $\hat{w}_i \hat{L}_i w_i L_i$. Exporter-side market clearing is therefore

$$\hat{w}_i \hat{L}_i w_i L_i + \beta_i = \sum_{j=1}^{N} \sum_{s=1}^{S} \hat{\pi}_{i,j,s}\, \pi_{i,j,s}\, \alpha_{j,s}\, \hat{w}_j \hat{L}_j w_j L_j, \qquad i = 1, \ldots, N. \tag{12}$$

The left-hand side is factor income of i after the wage and labor shocks, plus its fixed trade surplus. The right-hand side is the revenue it earns from all destinations and sectors, using baseline expenditure shares and their hats. Summing (12) over i and using $\sum_i \beta_i = 0$ implies

$$\sum_{i=1}^{N} \hat{w}_i \hat{L}_i w_i L_i = 1.$$

Thus, post-shock world income remains normalized to one.

Unit costs.

$$\hat{c}_{i,s} = \frac{\hat{w}_i}{\widehat{A}_{i,s}}.$$

Delivered prices.

$$\hat{P}_{i,j,s} = \frac{\hat{w}_i\, \widehat{\tau}_{i,j,s}}{\widehat{A}_{i,s}}.$$

Sectoral price index.

$$(\hat{P}_{j,s})^{1-\theta} = \sum_{i=1}^{N} \pi_{i,j,s} \, (\hat{P}_{i,j,s})^{1-\theta}.$$

Bilateral expenditure shares.

$$\hat{\pi}_{i,j,s} = \left(\frac{\hat{P}_{i,j,s}}{\hat{P}_{j,s}} \right)^{1-\theta} = \left(\frac{\hat{w}_i \, \widehat{\tau}_{i,j,s}}{\widehat{A}_{i,s} \, \hat{P}_{j,s}} \right)^{1-\theta}.$$

Aggregate CPI.

$$\hat{P}_j = \prod_{s=1}^{S} (\hat{P}_{j,s})^{\alpha_{j,s}}.$$

D Model with Input-Output Linkages

This appendix extends the baseline Armington model with Cobb-Douglas aggregation across sectors and CES aggregation within sectors to allow for intermediate-input use. The structure, notation, and timing follow the main text.

D.1 Countries and Sectors

There are N countries indexed by $i, j = 1, \dots, N$ and S sectors indexed by $s, r = 1, \dots, S$. A good produced in sector s of country i can be (i) absorbed as final demand in any country or (ii) used as an intermediate input by any sector in any country.

D.2 Preferences

Households in country j have Cobb-Douglas preferences over sectoral CES composites,

$$U_j = \prod_{s=1}^{S} \left(\frac{C_{j,s}}{\alpha_{j,s}} \right)^{\alpha_{j,s}}, \qquad \sum_{s=1}^{S} \alpha_{j,s} = 1,$$

where $C_{j,s}$ is final consumption of sector s in country j and $\alpha_{j,s}$ are expenditure shares. Let $P_{j,s}$ be the final-use price index for sector s in j and let P_j be the CPI. Optimal allocation implies

$$P_{j,s} C_{j,s} = \alpha_{j,s} P_j C_j, \qquad P_j = \prod_{s=1}^{S} P_{j,s}^{\alpha_{j,s}}.$$

World income is

$$Y \equiv \sum_{\ell=1}^{N} w_\ell L_\ell.$$

Trade imbalances are exogenous. Let β_j denote country j's trade surplus as a fraction of world income, with

$$\sum_{j=1}^{N} \beta_j = 0.$$

Final absorption in j is

$$P_j C_j = w_j L_j - \beta_j Y,$$

so $\beta_j > 0$ means that country j spends less than its income.

D.3 Technology and Costs

Production uses labor and sectoral intermediate-input composites. For country i, sector s, the unit cost is

$$c_{i,s} = \frac{v_{i,s}\, w_i^{\gamma_{i,s}} \prod_{r=1}^{S} \left(P^X_{i,r,s}\right)^{\eta_{i,r,s}(1-\gamma_{i,s})}}{A_{i,s}},$$

where:

- $A_{i,s}$ is productivity,
- $\gamma_{i,s} \in (0, 1]$ is the labor cost share,
- $\eta_{i,r,s} \geq 0$ is the cost share of input r in sector s of country i, with $\sum_{r=1}^{S} \eta_{i,r,s} = 1$,
- $P^X_{i,r,s}$ is the price index of intermediate inputs from sector r used in sector s of country i,
- $v_{i,s}$ is a cost shifter that is fixed across counterfactuals.

Thus, a fraction $\gamma_{i,s}$ of costs is labor and a fraction $(1 - \gamma_{i,s})$ is intermediates, allocated across input sectors according to $\eta_{i,r,s}$.

D.4 Trade Costs

Goods used for *final* demand face iceberg trade costs $\tau^F_{i,j,s} \geq 1$,

$$P^F_{i,j,s} = c_{i,s}\tau^F_{i,j,s} = \frac{v_{i,s}\, w_i^{\gamma_{i,s}} \prod_r \left(P^X_{i,r,s}\right)^{\eta_{i,r,s}(1-\gamma_{i,s})}}{A_{i,s}}\, \tau^F_{i,j,s}.$$

Goods used for *intermediate* demand face (possibly different) iceberg trade costs $\tau^X_{i,j,s,r} \geq 1$,

$$P^X_{i,j,s,r} = c_{i,s}\tau^X_{i,j,s,r}.$$

D.5 Final and Intermediate CES Aggregators

Within each sector s, final demand in country j is a CES composite of varieties produced in different countries,

$$C_{j,s} = \left(\sum_{i=1}^{N} \mu_{i,j,s}^{1/\theta}\, C_{i,j,s}^{(\theta-1)/\theta}\right)^{\theta/(\theta-1)}, \qquad \theta > 1,$$

with taste shifters $\mu_{i,j,s} > 0$. This implies the sectoral price index

$$P_{j,s} = \left(\sum_{i=1}^{N} \mu_{i,j,s} \left(P^F_{i,j,s} \right)^{1-\theta} \right)^{1/(1-\theta)}.$$

The corresponding final-expenditure share is

$$\pi^F_{i,j,s} \equiv \frac{P^F_{i,j,s} C_{i,j,s}}{\sum_{\ell=1}^{N} P^F_{\ell,j,s} C_{\ell,j,s}} = \frac{\mu_{i,j,s} \left(P^F_{i,j,s} \right)^{1-\theta}}{\sum_{\ell=1}^{N} \mu_{\ell,j,s} \left(P^F_{\ell,j,s} \right)^{1-\theta}}.$$

For intermediate use, I allow for a distinct set of taste weights $\mu^X_{i,j,s,r}$. The intermediate-input price index for inputs from sector s used by sector r in country j is

$$P^X_{j,s,r} = \left(\sum_{i=1}^{N} \mu^X_{i,j,s,r} \left(P^X_{i,j,s,r} \right)^{1-\theta} \right)^{1/(1-\theta)},$$

and the corresponding intermediate-expenditure share is

$$\pi^X_{i,j,s,r} = \frac{\mu^X_{i,j,s,r} \left(P^X_{i,j,s,r} \right)^{1-\theta}}{\sum_{\ell=1}^{N} \mu^X_{\ell,j,s,r} \left(P^X_{\ell,j,s,r} \right)^{1-\theta}}.$$

D.6 Gross Output

Let $Y_{i,s}$ denote gross output (revenue) of country i, sector s. Demand for $Y_{i,s}$ has two components.

Final demand. Country j spends $\alpha_{j,s}(w_j L_j - \beta_j Y)$ on sector s; exporter (i,s) gets the share $\pi^F_{i,j,s}$. Final-demand revenue of (i,s) is

$$\sum_{j=1}^{N} \pi^F_{i,j,s} \, \alpha_{j,s} \, (w_j L_j - \beta_j Y).$$

Intermediate demand. Country j, user sector r, spends a fraction $(1 - \gamma_{j,r})$ of its gross output $Y_{j,r}$ on intermediates; of that, the share $\eta_{j,s,r}$ is spent on inputs from sector s; and of that, exporter (i,s) receives the share $\pi^X_{i,j,s,r}$. Intermediate-demand revenue of (i,s) is

$$\sum_{j=1}^{N}\sum_{r=1}^{S}\pi^X_{i,j,s,r}\,(1-\gamma_{j,r})\,\eta_{j,s,r}\,Y_{j,r}.$$

Combining the two parts, gross output in country i and sector s satisfies

$$Y_{i,s}=\sum_{j=1}^{N}\pi^F_{i,j,s}\,\alpha_{j,s}\,(w_jL_j-\beta_jY)+\sum_{j=1}^{N}\sum_{r=1}^{S}\pi^X_{i,j,s,r}\,(1-\gamma_{j,r})\,\eta_{j,s,r}\,Y_{j,r}. \tag{13}$$

D.7 Labor Market Clearing

Labor income in country i is the labor-cost share across its sectors,

$$w_iL_i=\sum_{s=1}^{S}\gamma_{i,s}Y_{i,s}. \tag{14}$$

Summing (13) over i and s, and using

$$\sum_{i=1}^{N}\pi^F_{i,j,s}=1,\qquad \sum_{i=1}^{N}\pi^X_{i,j,s,r}=1,\qquad \sum_{s=1}^{S}\alpha_{j,s}=1,\qquad \sum_{s=1}^{S}\eta_{j,s,r}=1,\qquad \sum_{j=1}^{N}\beta_j=0,$$

I obtain

$$\begin{aligned}\sum_{i=1}^{N}\sum_{s=1}^{S}Y_{i,s}&=\sum_{j=1}^{N}\sum_{s=1}^{S}\alpha_{j,s}(w_jL_j-\beta_jY)+\sum_{j=1}^{N}\sum_{r=1}^{S}(1-\gamma_{j,r})Y_{j,r}\\&=\sum_{j=1}^{N}(w_jL_j-\beta_jY)+\sum_{j=1}^{N}\sum_{r=1}^{S}(1-\gamma_{j,r})Y_{j,r}\\&=\sum_{j=1}^{N}w_jL_j+\sum_{j=1}^{N}\sum_{r=1}^{S}(1-\gamma_{j,r})Y_{j,r}.\end{aligned}$$

The first term is world value added; the second term is total intermediate use.

D.8 Hat Algebra with Input–Output Linkages

Hats denote proportional changes relative to the baseline, e.g. $\hat{x}=x'/x$. I allow shocks to productivity, trade costs, and labor supply, so the objects $\widehat{A}_{i,s}$, $\widehat{\tau}^F_{i,j,s}$, $\widehat{\tau}^X_{i,j,r,s}$, and $\hat{L}_i$ can all differ from one. I keep $\{\beta_i\}$ fixed across counterfactuals, with $\sum_{i=1}^{N}\beta_i=0$, and I keep the taste shifters and cost shifters fixed across counterfactuals.

Throughout, s indexes the producing (and final-demand) sector. For intermediate inputs, r indexes the origin (input) sector and s indexes the destination (user) sector, so $\eta_{j,r,s}$ is the share of intermediate spending by sector s in country j allocated to inputs from sector r.

World income and final absorption. Baseline value added in country j is $w_j L_j$. Counterfactual world income is

$$Y' \equiv \sum_{\ell=1}^{N} w'_\ell L'_\ell = \sum_{\ell=1}^{N} \hat{w}_\ell \hat{L}_\ell \, w_\ell L_\ell.$$

Final absorption (nominal final expenditure) in country j is

$$P'_j C'_j = w'_j L'_j - \beta_j Y' = \hat{w}_j \hat{L}_j \, w_j L_j - \beta_j Y'.$$

Unit costs.

$$\hat{c}_{i,s} = \hat{w}_i^{\gamma_{i,s}} \prod_{r=1}^{S} \left(\hat{P}^X_{i,r,s}\right)^{(1-\gamma_{i,s})\eta_{i,r,s}} \widehat{A}_{i,s}^{-1}.$$

Delivered prices.

$$\hat{P}^F_{i,j,s} = \hat{c}_{i,s} \, \widehat{\tau}^F_{i,j,s}, \qquad \hat{P}^X_{i,j,r,s} = \hat{c}_{i,r} \, \widehat{\tau}^X_{i,j,r,s}.$$

Intermediate-input price indices. For inputs from sector r used by sector s in country j,

$$\left(\hat{P}^X_{j,r,s}\right)^{1-\theta} = \sum_{i=1}^{N} \pi^X_{i,j,r,s} \left(\hat{c}_{i,r} \, \widehat{\tau}^X_{i,j,r,s}\right)^{1-\theta}.$$

Final-use sectoral price indices.

$$\left(\hat{P}^F_{j,s}\right)^{1-\theta} = \sum_{i=1}^{N} \pi^F_{i,j,s} \left(\hat{c}_{i,s} \, \widehat{\tau}^F_{i,j,s}\right)^{1-\theta}.$$

Bilateral expenditure share changes. Final shares:

$$\hat{\pi}^F_{i,j,s} = \left(\frac{\hat{c}_{i,s} \, \widehat{\tau}^F_{i,j,s}}{\hat{P}^F_{j,s}}\right)^{1-\theta}, \qquad \pi^{F\prime}_{i,j,s} = \pi^F_{i,j,s} \hat{\pi}^F_{i,j,s}.$$

Intermediate shares:

$$\hat{\pi}^X_{i,j,r,s} = \left(\frac{\hat{c}_{i,r}\, \widehat{\tau}^X_{i,j,r,s}}{\hat{P}^X_{j,r,s}} \right)^{1-\theta}, \qquad \pi^{X\prime}_{i,j,r,s} = \pi^X_{i,j,r,s} \hat{\pi}^X_{i,j,r,s}.$$

Gross output. Let $Y'_{i,s}$ denote counterfactual gross output (revenue) of country i, sector s. Market clearing implies

$$Y'_{i,s} = \sum_{j=1}^{N} \pi^{F\prime}_{i,j,s}\, \alpha_{j,s} \left(\hat{w}_j \hat{L}_j\, w_j L_j - \beta_j Y' \right) + \sum_{j=1}^{N} \sum_{u=1}^{S} \pi^{X\prime}_{i,j,s,u}\, (1 - \gamma_{j,u})\, \eta_{j,s,u}\, Y'_{j,u}. \tag{15}$$

Equivalently, stacking $Y'_{i,s}$ into a vector y' over all (i, s), (15) can be written as

$$y' = f(\hat{w}, \hat{L}) + T(\hat{w}, \hat{L})\, y', \qquad y' = \left(I - T(\hat{w}, \hat{L}) \right)^{-1} f(\hat{w}, \hat{L}),$$

where f collects final-demand terms and T collects intermediate-demand coefficients.

Labor market clearing and wages. Labor market clearing implies

$$w'_i L'_i = \sum_{s=1}^{S} \gamma_{i,s} Y'_{i,s}.$$

Using $w'_i L'_i = \hat{w}_i \hat{L}_i\, w_i L_i$, the implied wage change is

$$\hat{w}_i = \frac{\sum_{s=1}^{S} \gamma_{i,s} Y'_{i,s}}{\hat{L}_i\, w_i L_i}.$$

Aggregate CPI.

$$\hat{P}_j = \prod_{s=1}^{S} \left(\hat{P}^F_{j,s} \right)^{\alpha_{j,s}}.$$

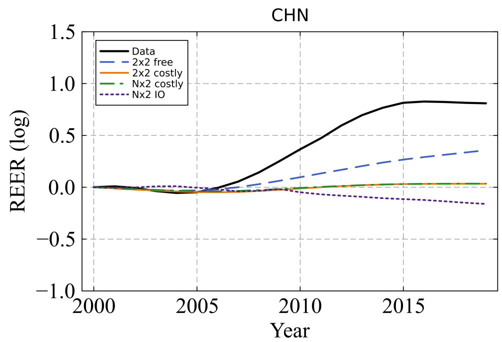
E Further Quantitative Results

E.1 By Country

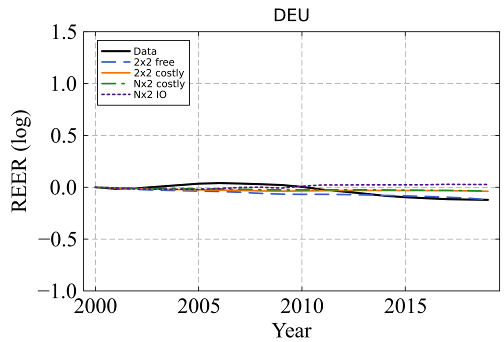
Figure A1 shows the results of REER by country over time.

Figure A1: REER Fits by country: Productivity Shock

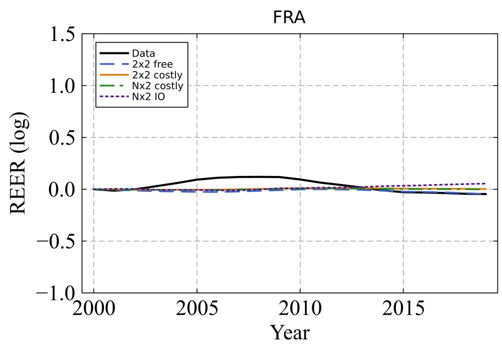
(a) China



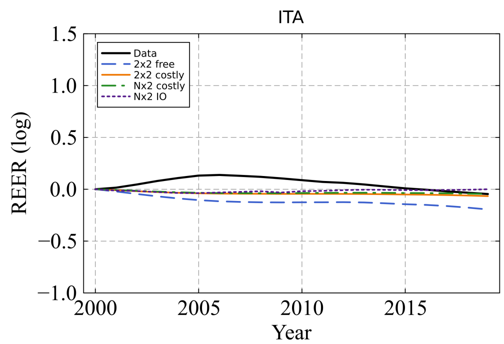
(b) Germany



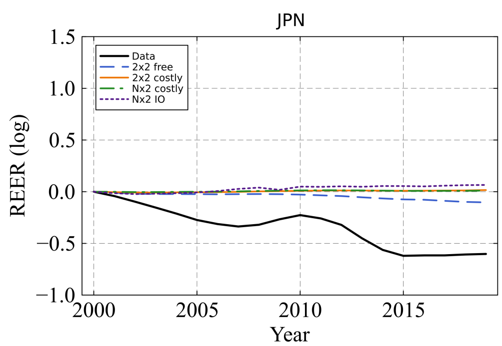
(c) France



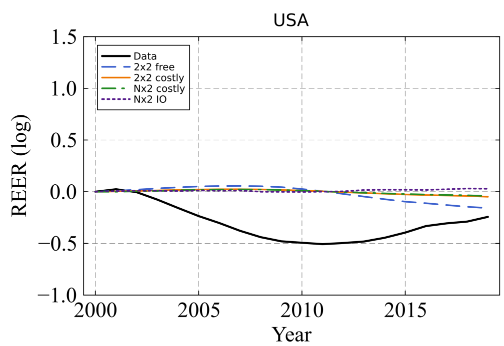
(d) Italy



(e) Japan



(f) U.S.



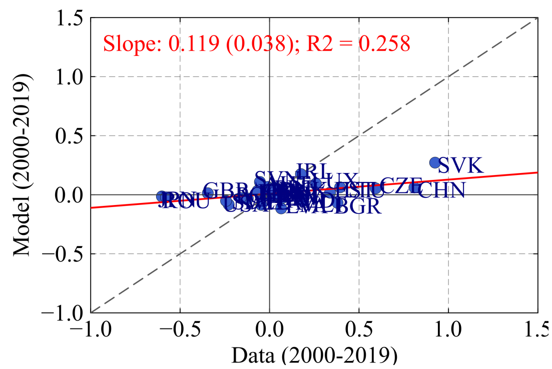
Notes: This figure compares the observed and model-generated REER paths across countries.

E.2 Adding Trade Cost Shocks

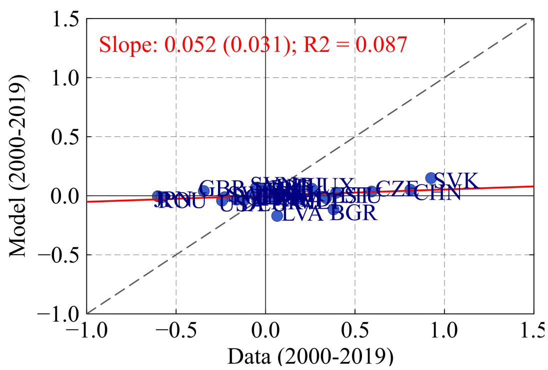
Figure A2 shows the results with time-varying trade costs. All the panels show that adding time-varying trade costs, calibrated as in Head and Ries (2001), does not help models' prediction on changes in REERs.

Figure A2: REER Long-Difference Fit vs. Data, 2000–2019; Adding Trade-Cost Shocks

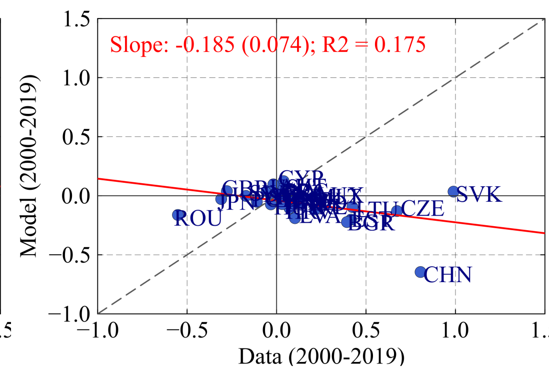
(a) 2×2 Costly Trade



(b) $N \times 2$ Costly Trade



(c) $N \times 2$ Costly Trade with IO Linkages



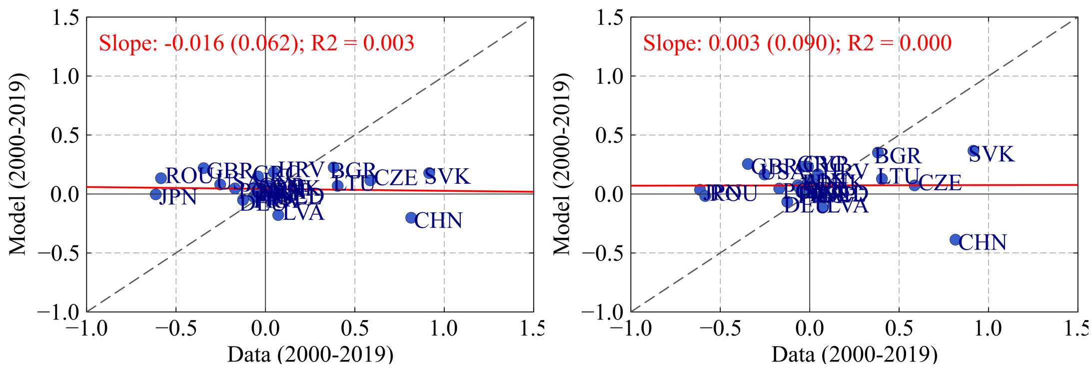
Notes: Each dot represents one country. The horizontal axis shows the HP-filtered change in log real effective exchange rate (REER) between 2000 and 2019 from the BIS data. The vertical axis shows the corresponding model-implied change when both bilateral iceberg trade costs $\{\tau_{i,j,s,t}\}$ and sectoral productivities $\{A_{i,s,t}\}$ evolve along their estimated low-frequency paths. Panels differ by model environment: (a) 2×2 with costly trade, (b) $N \times 2$ with costly trade, and (c) $N \times 2$ with costly trade and input–output linkages. A 45-degree line would indicate a perfect quantitative fit.

E.3 S (=23) sector models

Figure A3 shows the results under $N \times S$ models. Here, I use 23 sectors, including 13 goods sectors and 10 service sectors. Panel (a) shows the results without input-output linkages, and Panel (b) shows those with input-output linkages. Both panels show that having more detailed sectors in the models does not improve the models' performances to explain REER.

Figure A3: REER Long-Difference Fit vs. Data, 2000–2019; NxS models

(a) $N \times S$ Costly Trade (b) $N \times S$ Costly Trade with IO Linkages



Notes: Each dot represents one country. The horizontal axis shows the HP-filtered change in log real effective exchange rate (REER) between 2000 and 2019 from the BIS data. The vertical axis shows the corresponding model-implied change when sectoral productivities $\{A_{i,s,t}\}$ evolve along their estimated low-frequency paths. Panels differ by model environment: (a) $N \times S$ with costly trade, (b) $N \times S$ with costly trade and input–output linkages. A 45-degree line would indicate a perfect quantitative fit.